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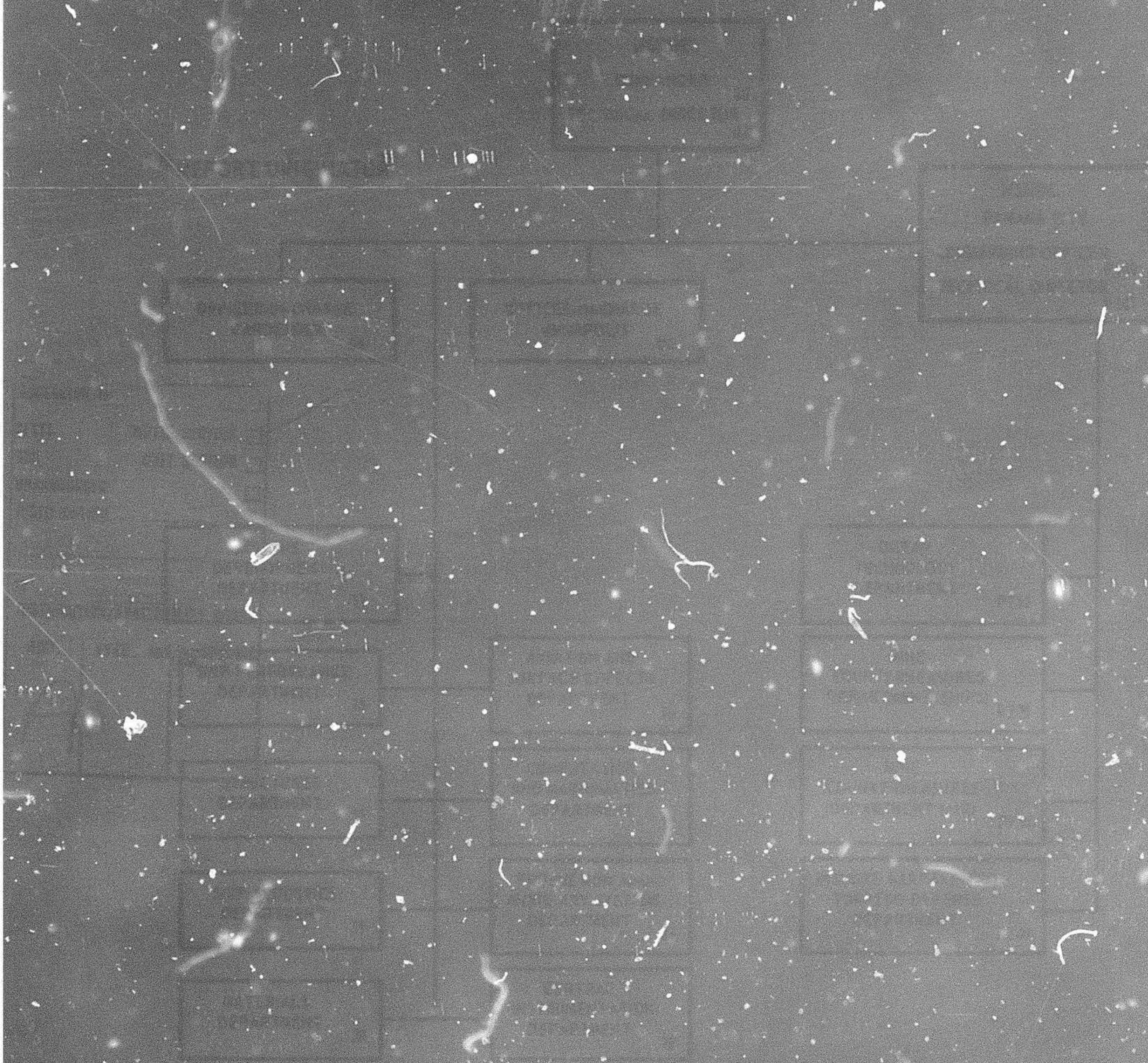

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DEPARTMENT OF THE NAVY

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SMALL-PERTURBATION ANALYSIS OF OSCILLATORY TOW-CABLE MOTION

by

Keith P. Kerney

November 1971 Report 3430

TABLE OF CONTENTS

LIST OF FIGURES

ABSTRACT

The equations of two-dimensional motion of a flexible inextensible cable are linearized by a small-perturbation approximation and sinusoidal time dependence is assumed. The simplified equations are integrated numerically by the Kutta-Merson method. Separate computer programs, OMWAY and OMFLO, have been written for the Quadrant I and Quadrant II cable-towed-body problems and are listed in the appendixes.

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INTRODUCTION

Accurate prediction of the motion of cable-towed-body systems is essential for their rational design and for simulation of their operation. For the simplest of these systems, a single body towed by a cable attached to a towing ship, adequate representations of the towpoint motion, the dynamic behavior of the cable as it is affected by the tension at its two ends and by hydrodynamic and gravitational forces, and the motion of the towed body are needed. If only steady-state motion is considered where the entire system is in steady rectilinear translation, analysis is possible by numerical integration of the differential equations that describe the cable configuration and tension. The case of steady-state motion of an inextensible tow cable in the plane described by gravity and the direction of translation has been solved numerically by Cuthill.[1] The tow speed and the tension and angle at the towed-body end of the cable are prescribed and the viscous force on the cable is represented by functions of cable angle based on experimental data. Integration of the two simultaneous first-order nonlinear ordinary differential equations for the cable tension and angle as functions of distance along the cable is performed by the Kutta-Merson method.

The unsteady two-dimensional motion of the same system was considered by Whicker.[2] Instead of prescribing tension and angle at the towed-body end of the cable, the equations of motion of the body become a boundary condition for the equations of motion of the cable. The cable equations are four first-order nonlinear partial

[1]References are listed on page 46.

differential equations for the tension, angle, and two velocity components as functions of time and distance along the cable. Unless the restriction to an inextensible cable is relaxed, the mathematical classification of these equations is parabolic since two are hyperbolic and two are parabolic. Consequently they cannot be solved by the method of characteristics.

The next section of this report describes the system of inextensible cable equations given in Reference 2 and a particular way of representing the viscous force on the cable. Then two important simplifying approximations are introduced: the unsteady dependent variables are assumed to be small perturbations of the steady-state variables, and their time dependence is assumed to be sinusoidal. The first of these permits the linearization of the unsteady equations; the second changes them from partial to ordinary differential equations with distance along the cable as independent variable. The digital computer programs used to integrate the equations are described in the next section and are listed in Appendixes B and C.

Thus the accuracy of the analysis reported here rests on the validity of four main assumptions: (1) that the motion of a real cable can be represented by the solution of the equations of motion of an inextensible cable, (2) that it remains in a vertical plane, (3) that it consists of small excursions from a steady-state configuration, and (4) that these excursions are sinusoidal in time.

EQUATIONS AND BOUNDARY CONDITIONS FOR TWO-DIMENSIONAL CABLE MOTION

The two-dimensional motion of an inextensible cable is described by four nonlinear first-order partial differential equations in which independent variables are distance along the cable s and time t and dependent variables are the cable angle ϕ, tension T, and normal and tangential velocity components U and V. For the case where the motion is in a plane containing gravity and ϕ is measured from the horizontal the equations are those derived in Appendix A which are

$$\frac{\partial U}{\partial s} - V\frac{\partial \phi}{\partial s} = -\frac{\partial \phi}{\partial t} \qquad [1]$$

$$U\frac{\partial \phi}{\partial s} + \frac{\partial V}{\partial s} = 0 \qquad [2]$$

$$\mu\left[(1+\lambda)\frac{\partial U}{\partial t} - V\frac{\partial \phi}{\partial t}\right] = -T\frac{\partial \phi}{\partial s} - F_L + w\cos\phi \qquad [3]$$

and

$$\mu\left[\frac{\partial V}{\partial t} + (1+\lambda)\,U\frac{\partial \phi}{\partial t}\right] = \frac{\partial T}{\partial s} - F_G - w \sin\phi \tag{4}$$

where μ is the cable mass per unit length, $\mu\lambda$ is its added mass (in water) per unit length, w is its weight in water per unit length, and F_L and F_G are the normal and tangential components of the viscous force per unit length acting on the cable. These equations were also derived by Whicker[2] for the case of $\lambda=0$.

Two cable-towed systems are considered in this report: in one a surface ship in a seaway is towing a deeply submerged body, and in the other a deeply submerged submarine is towing a float which is slightly beneath the free surface and subject to disturbances from its seaway. The two configurations are shown in Figures 1 and 2. In either configuration the (mean) towing direction is to the right and the origin of coordinates is at the center of mass of the towed body with x in the (mean) towing direction and y up. For the two configurations ϕ lies between 0 and $\pi/2$ and between $\pi/2$ and π so they are known as the Quadrant I and Quadrant II configurations respectively. The cable is of length L and the quadrant choice of ϕ causes the towpoint to be located at $s=L$ in Quadrant I and $s=-L$ in Quadrant II. More detailed discussion of these quadrant conventions is provided by Springston.[3]

In the Quadrant I problem the cable motion is excited by the effect of the seaway on the towing ship so the boundary conditions at $s=L$ are the motion of the towpoint; the boundary conditions at the bottom are provided by the equations of motion of the body tethered by the cable. In the Quadrant II problem the towpoint is on a deeply submerged submarine and the towed body is subject to time-varying forces due to the seaway. Therefore the boundary conditions at $s=0$ are the equations of motion of the tethered body subject to these forces; the other boundary conditions are that the point $s=-L$ be in steady rectilinear translation. Thus each problem has kinematic boundary conditions at one end and dynamic conditions at the other, and in either problem the excitation is due to the action of the seaway; this excitation is through the kinematic boundary conditions in Quadrant I and through the dynamic boundary conditions in Quadrant II.

In both problems the steady towing speed is c. In the Quadrant I problem the towpoint velocity components in surge and heave, positive forward and up, are $c + c_S$

(t) and c_h (t) and are assumed to be known. Therefore the kinematic boundary condition for Quadrant I is

$$U(L,t)\sin\phi(L,t) + V(L,t)\cos\phi(L,t) = c + c_s(t) \tag{5}$$

and

$$-U(L,t)\cos\phi(L,t) + V(L,t)\sin\phi(L,t) = c_h(t) \tag{6}$$

In the Quadrant II problem the towpoint is in steady horizontal translation at speed c so the kinematic boundary condition is

$$U(-L,t)\sin\phi(-L,t) + V(-L,t)\cos\phi(-L,t) = c \tag{7}$$

and

$$-U(-L,t)\cos\phi(-L,t) + V(-L,t)\sin\phi(-L,t) = 0 \tag{8}$$

The towed-body equations of motion used here are those of a very simple representation of the body dynamics. The body is assumed to be a point of mass m with added masses in surge and heave m_s and m_h, and weight (in water) W_B, on which known drag and lift forces D_B and L_B act. No body pitching is represented, any body motion which is excited is undamped and the mass center of the body, center of buoyancy, and point of cable attachment are coincident. In the Quadrant I problem D_B and L_B are taken as constants but in the Quadrant II problem they represent the exciting force from the seaway and are time-dependent. If U_B and V_B are the horizontal and vertical components of the body velocity, given by

$$U_B(t) = U(0,t)\sin\phi(0,t) + V(0,t)\cos\phi(0,t)$$

and

$$V_B(t) = -U(0,t)\cos\phi(0,t) + V(0,t)\sin\phi(0,t)$$

differentiation with respect to t and substitution into the body equations of motion leads to the dynamic boundary conditions

$$(m + m_t)\left\{\frac{\partial U}{\partial t}(0,t)\sin\phi(0,t) + \frac{\partial V}{\partial t}(0,t)\cos\phi(0,t) + [U(0,t)\cos\phi(0,t) - V(0,t)\sin\phi(0,t)]\frac{\partial\phi}{\partial t}(0,t)\right\} = -D_B \pm T(0,t)\cos\phi(0,t) \qquad (9)$$

and

$$(m + m_h)\left\{-\frac{\partial U}{\partial t}(0,t)\cos\phi(0,t) + \frac{\partial V}{\partial t}(0,t)\sin\phi(0,t) + [U(0,t)\sin\phi(0,t) + V(0,t)\cos\phi(0,t)]\frac{\partial\phi}{\partial t}(0,t)\right\} = L_B - W_B \pm T(0,t)\sin\phi(0,t) \qquad (10)$$

with the signs in front of the $T(o,t)$ chosen as plus for the Quadrant I problem and minus for the Quadrant II problem.

In steady-state towing-cable theory the viscous force components F_L and F_G are assumed to be equal to D times known functions of ϕ, where D is the drag per unit length of the cable when ϕ is $\pi/2$. F_L/D and F_G/D are known as the normal and tangential loading functions. In this report it is assumed that the viscous force on the cable in unsteady flow can be represented by an unsteady generalization of the steady-state force. D is expressed in terms of C_D, the coefficient of steady-state drag per unit length when the cable is normal to the flow, as $D=\frac{\rho c^2}{2}hC_D$ where ρ is the water density and h is the cable thickness, or dimension in the direction normal to the plane in which the motion lies. The unsteady generalization of this expression for D consists of assuming that D is given by

$$D = \rho\,\frac{U^2 + V^2}{2}\,hC_D \qquad (11)$$

where C_D remains the drag coefficient for the cable normal to a steady flow but U and V are instantaneous values of the velocity components. The unsteady generalization of the loading functions consists of replacing the functional dependence on ϕ with the same functional dependence on θ, defined by $\theta=\tan^{-1} U/V$ since, as can be seen in Figures 1 and 2, this is the angle between the tangent to the cable and the direction of cable motion. (θ clearly reduces to ϕ when U and V take on their

steady-state values of $U = c \sin \phi$ and $V = c \cos \phi$.) Comparison of Figure 2 with Figure 1 shows that the positive viscous force components in Quadrant II, F_L and $-F_G$, should exhibit the same functional dependence on π-θ, the acute angle between the cable tangent and the flow direction, that F_L and F_G exhibit on θ in Quadrant I. Since the generalized loading functions proposed in Reference 3 are used in this report, this requirement will be satisfied if the loading functions are given by

$$\frac{F_L}{D} = A_o^{(\Lambda)} \pm A_1^{(\Lambda)} \cos \theta + A_2^{(\Lambda)} \cos 2\theta + B_1^{(\Lambda)} \sin \theta \pm B_2^{(\Lambda)} \sin 2\theta \qquad (12)$$

and

$$\frac{F_G}{D} = \pm A_o^{(\Gamma)} + A_1^{(\Gamma)} \cos \theta \pm A_2^{(\Gamma)} \cos 2\theta \pm B_1^{(\Gamma)} \sin \theta + B_2^{(\Gamma)} \sin 2\theta \qquad (13)$$

where plus signs are used for Quadrant I and minus signs for Quadrant II. A and B are constants to be determined.

An example of how A and B may be related to experimental data is provided by Eames,[4] who suggests that the viscous force on the cable can be assumed to consist of a pressure drag, $D(1\text{-}f) \sin^2 \theta$, which acts normal to the cable tangent and a frictional drag, Df, which acts parallel to the flow direction or horizontal for the steady-state case. Consequently $F_L = D(1\text{-}f) \sin^2 \theta + Df \sin \theta$ and $F_G = Df \cos \theta$, and A and B are given by $A_0^{(\Lambda)} = \frac{1-f}{2}$, $A_2^{(\Lambda)} = -\frac{1-f}{2}$, $B_1^{(\Lambda)} = f$, $A_1^{(\Gamma)} = f$ and the other six equal to zero. This representation results in reasonable values at the ends of the range of θ; for $\theta = \pi/2$, F_L is D and F_G is zero, for $\theta = 0$, F_L is zero and F_G is Df. Furthermore only two quantities, C_D and f, need to be determined from experiments. They should be expected to be dependent on Reynolds number.

The parameter λ, the ratio of the added mass per unit length in water to mass per unit length for motion in the x-y plane, will be represented in the form

$$\lambda = \frac{\rho h^2 \pi}{4\mu} \nu \qquad (14)$$

where ν is a constant to be determined. Ideal-fluid theory predicts $\nu=1$ for bare round cables.

In either quadrant the coordinates x and y of a point on the cable are related to s and ϕ by $\frac{\partial x}{\partial s} = \cos\phi$ and $\frac{\partial y}{\partial s} = \sin\phi$ so they are given by

$$x(s,t) = \int_o^s \cos\phi\,(s',t)\,ds' \qquad [15]$$

and

$$y(s,t) = \int_o^s \sin\phi\,(s',t)\,ds' \qquad [16]$$

To summarize, U, V, ϕ and T are given as functions of s and t by solutions of Equations (1)-(4), with loading functions given by Equations [12] and [13] (with appropriate signs), D given by Equation [11], λ given by Equation [14], kinematic boundary conditions provided by Equations [5] and [6] or [7] and [8], and dynamic boundary conditions by [9] and [10] (with appropriate signs). Then x and y can be found using Equations [15] and [16].

SMALL-PERTURBATION FREQUENCY-DOMAIN EQUATIONS AND THEIR SOLUTIONS

Approximate solutions to the system of equations given in the previous section are obtained for the case when the exciting disturbance - the towpoint motion in the Quadrant I problem and the time-varying force in the Quadrant II problem - is small and sinusoidal in time by assuming that ϕ, T, U, and V are equal to the steady-state values plus a small perturbation term which is proportional to the cosine of ωt minus a phase angle. ω is constant and the magnitude and phase angle of the perturbation terms are functions of s found as solutions of eight linear ordinary differential equations obtained from the four nonlinear partial differential equations, [1]-[4]. Since the solutions are the magnitudes and phases of a sinusoidal oscillation they are known as frequency-domain solutions.

The steady-state solutions are denoted by terms with subscript zero. For example $\phi(s,t)$ is given by

$$\begin{aligned}\phi(s,t) &= \phi_0(s) + \phi_M(s)\cos[\omega t - \delta_\phi(s)] + \ldots \\ &= \phi_0(s) + \phi_M(s)\,[\cos\omega t\cos\delta_\phi(s) + \sin\omega t\sin\delta_\phi(s)] + \ldots \\ &= \phi_0(s) + \phi_R(s)\cos\omega t + \phi_I(s)\sin\omega t + \ldots \\ &= \phi_0(s) + \mathcal{R}e\,[\phi_1(s)e^{-i\omega t}] + \ldots\end{aligned} \qquad [17]$$

where Re means that the real part is to be taken, i is the imaginary unit, and $+\ldots$ indicates that $\phi_M(s)\cos[\omega t - \delta_\phi(s)]$ is the leading term in a perturbation series for $\phi(s,t) - \phi_0(s)$. ϕ_R and ϕ_I are related to the magnitude and phase ϕ_M and δ_ϕ by

$$\phi_R = \phi_M \cos \delta_\phi$$

and

$$\phi_I = \phi_M \sin \delta_\phi$$

so

$$\phi_M = \sqrt{\phi_R^{\,2} + \phi_I^{\,2}}$$

and

$$\delta_\phi = \tan^{-1} \frac{\phi_I}{\phi_R}$$

ϕ_1 is given by $\phi_1 = \phi_R + i\phi_I$.

Similarly

$$\begin{aligned} T(s,t) &= T_0(s) + T_M(s)\cos[\omega t - \delta_T(s)] + \ldots \\ &= T_0(s) + Re\,[T_1(s)e^{-i\omega t}] + \ldots \end{aligned} \qquad [18]$$

with corresponding relations between T_M, δ_T, T_R, T_I, and T_1. As was seen in the previous section, U_0 and V_0 are given by

$$U_0(s) = c \sin \phi_0(s) \qquad [19]$$

and

$$V_0(s) = c \cos \phi_0(s) \qquad [20]$$

and satisfy Equations [1] and [2]. It is convenient to represent U in the form

$$\begin{aligned} U(s,t) &= c \sin\left\{\phi_0(s) + Re[\phi_1(s)e^{-i\omega t}]\right\} + Re[U_1(s)e^{-i\omega t}] + \ldots \\ &= c \sin \phi_0(s) + c \cos \phi_0(s)\, Re[\phi_1(s)e^{-i\omega t}] + Re[U_1(s)e^{-i\omega t}] + \ldots \end{aligned}$$

so the expansion for U is

$$U(s,t) = c \sin \phi_0(s) + \mathcal{Re}\left\{ [c\phi_1(s) \cos \phi_0(s) + U_1(s)] e^{i\omega t} \right\} + \ldots \qquad [21]$$

In the same way

$$V(s,t) = c \cos \phi_0(s) + \mathcal{Re}\left\{ [-c\phi_1(s) \sin \phi_0(s) + V_1(s)] e^{i\omega t} \right\} + \ldots \qquad [22]$$

Then if $U_1 = U_R + iU_I$ and U_M and δ_U are given by

$$U_M = \sqrt{(c\phi_R \cos \phi_0 + U_R)^2 + (c\phi_I \cos \phi_0 + U_I)^2}$$

and

$$\delta_U = \tan^{-1} \frac{c\phi_I \cos \phi_0 + U_I}{c\phi_R \cos \phi_0 + U_R}$$

Equation [21] reduces to

$$U(s,t) = U_0(s) + U_M(s) \cos [\omega t - \delta_U(s)] + \ldots$$

Similarly, $V_1 = V_R + iV_I$ and V_M and δ_V are given by

$$V_M = \sqrt{(-c\phi_R \sin \phi_0 + V_R)^2 + (-c\phi_I \sin \phi_0 + V_I)^2}$$

and

$$\delta_V = \tan^{-1} \frac{-c\phi_I \sin \phi_0 + V_I}{-c\phi_R \sin \phi_0 + V_R}$$

to give

$$V(s,t) = V_0(s) + V_M(s) \cos [\omega t - \delta_V(s)] + \ldots$$

In the Quadrant I problem the excitation is due to speeds in surge and heave at the towpoint, $c_s(t)$ and $c_h(t)$, yet it is desirable to have prescribed displacements in surge and heave, $a_s(t)$ and $a_h(t)$, as inputs.

These are given by

$$a_s(t) = a_{sM} \cos (\omega t - \delta_s)$$

and

$$a_h(t) = a_{hM} \cos(\omega t - \delta_h)$$

so c_S is given by

$$\begin{aligned} c_s(t) &= -\omega a_{sM} \sin(\omega t - \delta_s) \\ &= -\omega a_{sM} (\sin \omega t \cos \delta_s - \cos \omega t \sin \delta_s) \\ &= c_{sR} \cos \omega t + c_{sI} \sin \omega t \\ &= \mathcal{R}e\,(c_{s1} e^{-i\omega t}) \end{aligned} \qquad [23]$$

where $c_{s1} = c_{sR} + ic_{sI}$ with $c_{sR} = \omega a_{sM} \sin \delta_s$ and $c_{sI} = -\omega a_{sM} \cos \delta_s$. Similarly, c_h is given by

$$c_h(t) = \mathcal{R}e\,(c_{h1} e^{-i\omega t}) \qquad [24]$$

where $c_{h1} = c_{hR} + ic_{hI}$, $c_{hR} = \omega a_{hM} \sin \delta_h$, and $c_{hI} = -\omega a_{hM} \cos \delta_h$

The excitation for the Quadrant II problem is due to time-varying drag and lift forces on the towed body. Therefore the forces are given by

$$\begin{aligned} D_B &= D_{B0} + D_M \cos(\omega t - \delta_D) \\ &= D_{B0} + \mathcal{R}e\,(D_1 e^{-i\omega t}) \end{aligned} \qquad [25]$$

and

$$\begin{aligned} L_B &= L_{B0} - W_B + L_M \cos(\omega t - \delta_L) \\ &= L_{B0} - W_B + \mathcal{R}e\,(L_1 e^{-i\omega t}) \end{aligned} \qquad [26]$$

where D_1 and L_1 are zero for the Quadrant I problem and given by $D_1 = D_R + iD_I$ and $L_1 = L_R + iL_I$ in the Quadrant II problem where $D_R = D_M \cos \delta_D$, $D_I = D_M \sin \delta_D$, $L_R = L_M \cos \delta_L$, and $L_I = L_M \sin \delta_L$.

Substitution of Equations [21] and [22] into Equation [11] gives

$$\begin{aligned} D &= \frac{\rho h C_D}{2} \Big\{ c^2 + 2c \sin \phi_0 \,\mathcal{R}e\,[(c\phi_1 \cos \phi_0 + U_1) e^{-i\omega t}] + \ldots \\ &\qquad + 2c \cos \phi_0 \,\mathcal{R}e\,[(-c\phi_1 \sin \phi_0 + V_1) e^{-i\omega t}] + \ldots \Big\} \\ &= D_0 \left\{ 1 + \frac{2}{c} \mathcal{R}e\,[c_1(s) e^{-i\omega t}] \right\} + \ldots \end{aligned} \qquad [27]$$

where $D_0 = (\rho/2) c^2 h C_D$ and $c_1 = U_1 \sin \phi_0 + V_1 \cos \phi_0$.

Equations [21] and [22] give

$$\frac{U}{V} = \frac{c \sin \phi_0 + \mathcal{R}e\,[(c\phi_1 \cos \phi_0 + U_1)e^{-i\omega t}] + \ldots}{c \cos \phi_0 + \mathcal{R}e\,[(-c\phi_1 \sin \phi_0 + V_1)e^{-i\omega t}] + \ldots}$$

$$= \tan \phi_0 \; \frac{1 + \mathcal{R}e\left(\dfrac{c\phi_1 \cos \phi_0 + U_1}{c \sin \phi_0}\, e^{-i\omega t}\right) + \ldots}{1 + \mathcal{R}e\left(\dfrac{-c\phi_1 \sin \phi_0 + V_1}{c \cos \phi_0}\, e^{-i\omega t}\right) + \ldots}$$

$$= \tan \phi_0 \left\{ 1 + \mathcal{R}e\left[\left(\frac{c\phi_1 \cos \phi_0 + U_1}{c \sin \phi_0} + \frac{c\phi_1 \sin \phi_0 - V_1}{c \cos \phi_0}\right) e^{-i\omega t}\right] + \ldots \right\}$$

Therefore

$$\theta = \tan^{-1}\left\{ \tan \phi_0 + \tan \phi_0 \, \mathcal{R}e\left[\left(\frac{c\phi_1 \cos \phi_0 + U_1}{c \sin \phi_0} + \frac{c\phi_1 \sin \phi_0 - V_1}{c \cos \phi_0}\right) e^{-i\omega t}\right] + \ldots \right\}$$

and use of the Taylor expansion for the inverse tangent gives

$$\theta = \phi_0 + \frac{\tan \phi_0}{1 + \tan^2 \phi_0}\, \mathcal{R}e\left[\left(\frac{c\phi_1 \cos\phi_0 + U_1}{c \sin \phi_0} + \frac{c\phi_1 \sin\phi_0 - V_1}{c \cos \phi_0}\right) e^{-i\omega t}\right] + \ldots \qquad [28]$$

$$= \phi_0 + \mathcal{R}e\,[\theta_1(s)e^{-i\omega t}] + \ldots$$

where

$$\theta_1 = \frac{\tan \phi_0}{1 + \tan^2 \phi_0}\left(\frac{c\phi_1 \cos \phi_0 + U_1}{c \sin \phi_0} + \frac{c\phi_1 \sin \phi_0 - V_1}{c \cos \phi_0}\right)$$

$$= \phi_1 + \frac{U_1}{c} \cos\phi_0 - \frac{V_1}{c} \sin \phi_0$$

Substitution of Equations [27] and [28] into Equations [12] and [13] shows that the expansions for the viscous-force components are

$$F_L = F_L\,(D_0,\phi_0) + 2\,F_L\,(D_0,\phi_0)\,\mathcal{R}e\left[\frac{c_1(s)}{c}\, e^{-i\omega t}\right] + \frac{dF_L}{d\phi_0}\,(D_0,\phi_0)\,\mathcal{R}e\,[\theta_1(s)e^{-i\omega t}] + \ldots \qquad [29]$$

and

$$F_G = F_G\,(D_0,\phi_0) + 2\,F_G\,(D_0,\phi_0)\,\mathcal{R}e\left[\frac{c_1(s)}{c}\, e^{-i\omega t}\right] + \frac{dF_G}{d\phi_0}\,(D_0,\phi_0)\,\mathcal{R}e\,[\theta_1(s)e^{-i\omega t}] + \ldots \qquad [30]$$

The two steady-state cable equations are obtained by substituting the steady-state terms in the expansions for ϕ, T, U, V, F_L, and F_G given by Equations [17]

through [22], [29], and [30] into Equations [3] and [4].
Thus

$$-T_0 \frac{d\phi_0}{ds} - F_L\ (D_0,\phi_0) + w \cos \phi_0 = 0 \qquad [31]$$

and

$$\frac{dT_0}{ds} - F_G\ (D_0,\phi_0) - w \sin \phi_0 = 0 \qquad [32]$$

Boundary conditions at $s = 0$ are found by substitution of these steady-state terms into Equations [9] and [10]. The result is

$$-D_{BO} \pm T_0\ (0) \cos \phi_0\ (0) = 0$$

and

$$L_{BO} - W_B \pm T_0\ (0) \sin \phi_0 (0) = 0$$

or

$$T_0\ (0) = \sqrt{D_{BO}{}^2 + (L_{BO} - W_B)} \qquad [33]$$

and

$$\phi_0\ (0) = \tan^{-1} \frac{L_{BO} - W_B}{-D_{BO}} \qquad [34]$$

D_{BO} is positive so Equation [34] shows that W_B must be greater then L_{BO} for the Quadrant I problem and less than L_{BO} for the Quadrant II problem.

Equations [31] and [32], which are simultaneous nonlinear first-order ordinary differential equations for ϕ_0 and T_0, are integrated numerically on $0 < s \leq L$ in Quadrant I and $0 > s \geq -L$ in Quadrant II, with initial conditions provided by Equations [33] and [34]. The integration is performed by the same Kutta-Merson subroutine, KUTMER, that is used in Reference 1. Thus the results of Reference 1 are duplicated although it is necessary to write separate programs for Quadrants I and II and the loading functions are restricted to those that can be described by Equations [12] and [13].

Substitution of Equations [17] – [22], [29], and [30] into Equations [1] –

[4], use of small-angle approximations to the multiple-angle formulas, subtraction of Equations [31] and [32], and cancellation of a mutual factor $e^{-i\omega t}$ result in

$$\frac{dU_1}{ds} - V_1 \frac{d\phi_0}{ds} = i\omega\phi_1 \tag{35}$$

$$U_1 \frac{d\phi_0}{ds} + \frac{dV_1}{ds} = 0 \tag{36}$$

$$\begin{aligned} -i\omega\mu\,[\lambda c\phi_1 \cos\phi_0 + (1+\lambda)\,U_1] = -T_0 \frac{d\phi_1}{ds} - T_1 \frac{d\phi_0}{ds} - \frac{2}{c} F_L\,(D_0,\phi_0)\,c_1 \\ - \frac{dF_L}{d\phi_0}\,(D_0,\phi_0)\,\theta_1 - w\phi_1 \sin\phi_0 \end{aligned} \tag{37}$$

and

$$-i\omega\mu\,(V_1 + \lambda c\phi_1 \sin\phi_0) = \frac{dT_1}{ds} - \frac{2}{c} F_G\,(D_0,\phi_0)\,c_1 - \frac{dF_G}{d\phi_0}\,(D_0,\phi_0)\,\theta_1 - w\phi_1 \cos\phi_0 \tag{38}$$

Since ϕ_0 and T_0 are given by numerical integration of Equations [31] and [32] and appear as coefficients in Equations [35] through [38], these four linear ordinary differential equations must be integrated numerically. This is also done by the subroutine KUTMER.

Boundary conditions at $s = 0$ are found by substitution of Equations [17]–[22] into [9] and [10]. Thus

$$\begin{aligned} -i\omega(m+m_s)[U_1(0)\sin\phi_0(0) + V_1(0)\cos\phi_0(0)] = -D_1 \pm [T_1(0)\cos\phi_0(0) \\ - T_0(0)\,\phi_1(0)\sin\phi_0(0)] \end{aligned}$$

and

$$\begin{aligned} -i\omega(m+m_h)[-U_1(0)\cos\phi_0(0) + V_1(0)\sin\phi_0(0)] = L_1 \pm [T_1(0)\sin\phi_0(0) \\ + T_0(0)\,\phi_1(0)\cos\phi_0(0)] \end{aligned}$$

or

$$\begin{aligned} T_1(0) = \pm\,(-i\omega\{U_1(0)(m_s - m_h)\sin\phi_0(0)\cos\phi_0(0) + V_1(0)[m + m_s\cos^2\phi_0(0) \\ + m_h \sin^2\phi_0(0)]\} + D_1\cos\phi_0(0) - L_1\sin\phi_0(0)) \end{aligned} \tag{39}$$

and

$$\phi_1(0) = \frac{\pm 1}{T_0(0)} \left(-i\omega\left\{-U_1(0)[m + m_s \sin^2\phi_0(0) + m_h \cos^2\phi_0(0)] - V_1(0)(m_s - m_h)\right.\right.$$
$$\left.\left.\sin\phi_0(0)\cos\phi_0(0)\right\} - D_1 \sin\phi_0(0) - L_1 \cos\phi_0(0)\right) \qquad [40]$$

with plus signs for Quadrant I and minus signs for Quadrant II. Recall that D_1 and L_1 are zero for the Quadrant I problem.

Boundary conditions at $s = L$ for the Quadrant I problem are obtained by substitution of Equations [17] and [21] – [24] into Equations [5] and [6]. Thus

$$U_1(L) = c_{s1} \sin\phi_0(L) - c_{h1} \cos\phi_0(L) \qquad [41]$$

and

$$V_1(L) = c_{s1} \cos\phi_0(L) + c_{h1} \sin\phi_0(L) \qquad [42]$$

Substitution of Equations [17], [21], and [22] into Equations [7] and [8] shows that the boundary conditions at $s = -L$ in the Quadrant II problem are

$$U_1(-L) = 0 \qquad [43]$$

and

$$V_1(-L) = 0 \qquad [44]$$

Satisfaction of the two-point boundary conditions is achieved by assigning different sets of initial values to the dependent variables at the lower, or passive, end of the cable and integrating Equations [35]–[38] up the cable twice, once with each set of initial values, so that two different solutions are obtained along the cable. Then the linearity and homogeneity of the lower boundary conditions and the equations permit linear superposition of the two solutions such that the inhomogeneous boundary conditions at the upper, or excited, end are satisfied. This is known as a "shooting" method of solving a two-point boundary-value problem.

The two sets of solutions are called the A-mode and B-mode solutions and the solution to the full problem is given by

$$U_1 = C_A\, U_{1A} + C_B\, U_{1B} \qquad [45]$$

$$V_1 = C_A\, V_{1A} + C_B\, V_{1B} \qquad [46]$$

$$\phi_1 = C_A \phi_{1A} + C_B \phi_{1B} \qquad [47]$$

and

$$T_1 = C_A T_{1A} + C_B T_{1B} \qquad [48]$$

where the subscripts on U_1, V_1, ϕ_1, and T_1 denote the solution mode and C_A and C_B are constants to be determined below.

In the Quadrant I problem, values of $U_{1A}(0)$, $U_{1B}(0)$, $V_{1A}(0)$ and $V_{1B}(0)$ are assigned and $T_{1A}(0)$, $T_{1B}(0)$, $\phi_{1A}(0)$ and $\phi_{1B}(0)$ are found from Equations [39] and [40] with plus signs and $D_1 = L_1 = 0$. Next Equations [35]–[38] are integrated on $0 < s < L$ so that both modal solutions are known on $0 < s < L$. Then Equations [45], [46], [41], and [42] show that C_A and C_B must be given values such that

$$C_A U_{1A}(L) + C_B U_{1B}(L) = c_{s1} \sin \phi_0(L) - c_{h1} \cos\phi_0(L)$$

and

$$C_A V_{1A}(L) + C_B V_{1B}(L) = c_{s1} \cos\phi_0(L) + c_{h1} \sin\phi_0(L)$$

are satisfied. Therefore

$$C_A = \frac{[c_{s1} \sin\phi_0(L) - c_{h1} \cos\phi_0(L)]\, V_{1B}(L) - [c_{s1} \cos\phi_0(L) + c_{h1} \sin\phi_0(L)]\, U_{1B}(L)}{U_{1A}(L)\, V_{1B}(L) - U_{1B}(L)\, V_{1A}(L)} \qquad 49]$$

and

$$C_B = \frac{[c_{s1} \cos\phi_0(L) + c_{h1} \sin\phi_0(L)]\, U_{1A}(L) - [c_{s1} \sin\phi_0(L) - c_{h1} \cos\phi_0(L)]\, V_{1A}(L)}{U_{1A}(L)\, V_{1B}(L) - U_{1B}(L)\, V_{1A}(L)} \qquad [50]$$

Finally U_1, V_1, ϕ_1, and T_1 are found on $0 < s < L$ by substituting back into Equations [45]–[48].

The Quadrant II problem is solved in much the same way. Values are assigned to $\phi_{1A}(-L)$, $\phi_{1B}(-L)$, $T_{1A}(-L)$, and $T_{1B}(-L)$. Then Equations [43] and [44] show that $U_{1A}(-L)$, $U_{1B}(-L)$, $V_{1A}(-L)$ and $V_{1B}(-L)$ are all zero. (Otherwise Equations [45] and [46], with left-hand sides equal to zero, show that C_A and C_B would be determined by conditions at $s = -L$). Equadtions [35]–[38] arc integrated on $-L < s < 0$ so that both modal solutions are known on $-L < s < 0$. Equations [45]–[48] and [39] and [40] with minus signs show that, if P and Q are defined by

$$P = T_1(0) - i\omega \left\{ U_1(0)(m_s - m_h)\cos\phi_0(0)\sin\phi_0(0) + V_1(0)[m + m_s\cos^2\phi_0(0) + m_h\sin^2\phi_0(0)] \right\}$$

and

$$Q = T_0(0)\phi_1(0) - i\omega \left\{ U_1(0)[m + m_s\sin^2\phi_0(0) + m_h\cos^2\phi_0(0)] - V_1(0)(m_s - m_h)\cos\phi_0(0)\sin\phi_0(0) \right\}$$

with P_A, P_B, Q_A, and Q_B defined with appropriate modal values of T_1, ϕ_1, U_1, and V_1 on the right-hand side, the boundary conditions at $s = 0$ become

$$C_A P_A + C_B P_B = -D_1\cos\phi_0(0) + L_1\sin\phi_0(0)$$

and

$$C_A Q_A + C_B Q_B = D_1\sin\phi_0(0) + L_1\cos\phi_0(0)$$

Therefore C_A and C_B are given by

$$C_A = \frac{[-D_1\cos\phi_0(0) + L_1\sin\phi_0(0)]\,Q_B - [D_1\sin\phi_0(0) + L_1\cos\phi_0(0)]\,P_B}{P_A Q_B - P_B Q_A} \quad [51]$$

and

$$C_B = \frac{[D_1\sin\phi_0(0) + L_1\cos\phi_0(0)]\,P_A - [-D_1\cos\phi_0(0) + L_1\sin\phi_0(0)]\,Q_A}{P_A Q_B - P_B Q_A} \quad [52]$$

Finally U_1, V_1, ϕ_1, and T_1 are found on $-L < s < 0$ by substituting back into Equation [45]–[48].

The analysis is completed by computing the cable configuration. This is denoted by x and y and is found from Equations [15] and [16]. Equation [17] suggests that x and y are given by expansions of the form

$$x(s,t) = x_0(s) + \mathcal{Re}\,[x_1(s)e^{-i\omega t}] + \ldots \qquad [53]$$

and

$$y\,(s,t) = y_0(s) + \mathcal{Re}\,[y_1(s)e^{-i\omega t}] + \ldots \qquad [54]$$

Then if $x_1 = x_R + ix_I$, $x_M = \sqrt{x_R^{\,2} + x_I^{\,2}}$, and $\delta_x = \tan^{-1}\dfrac{x_I}{x_R}$ it is found that $x = x_0 + x_M \cos\,(wt - \delta_x) + \ldots$

Similarly $y_1 = y_R + iy_I$, $y_M = \sqrt{y_R^{\,2} + y_I^{\,2}}$, and $\delta_y = \tan^{-1}\dfrac{y_I}{y_R}$

Substitution of Equations [17], [53], and [54] into Equations [15] and [16] and use of the small-angle approximations to the multiple-angle formulas give

$$x_0 = \int_0^s \cos\phi_0(s')\,ds' \qquad [55]$$

$$x_1 = -\int_0^s \phi_1(s')\,\sin\phi_0(s')\,ds' \qquad [56]$$

$$y_0 = \int_0^s \sin\phi_0(s')\,ds' \qquad [57]$$

and

$$y_1 = \int_0^s \phi_1(s')\,\cos\phi_0(s')\,ds' \qquad [58]$$

These integrals are to be evaluated for $0 < s < L$ in the Quadrant I problem and for $0 > s > -L$ in the Quadrant II problem. Because of the choice of origin, the configuration given is that seen by an observer moving with the towed body.

After the complex-valued functions ϕ_1, T_1, $c\phi_1 \cos\phi_0 + U_1$, $-c\phi_1 \sin\phi_0 + V_1$, x_1, and y_1 have been found the amplitudes and phases of the perturbation quantities are easily found.

There is one redundancy in this small-perturbation frequency-domain solution to

the two-dimensional cable problem since there is no way of assigning a meaning to the time $t = 0$. Consequently the phase angle of either the horizontal or vertical excitation is meaningless and may be set equal to zero.

COMPUTER PROGRAMS

The digital computer programs which solve the Quadrant I and Quadrant II problems are OMWAY and OMFLO respectively, and are listed in Appendixes B and C, together with sample calculations. They are written in FORTRAN IV for a CDC 6700 computer.

Although Equations [35]–[38], [53], [54], and the boundary conditions are written for complex-valued functions of s, they must be separated into real and imaginary parts because the integrating subroutine KUTMER is not available in complex arithmetic. However, the existence of a complex-conjugate relationship between the real and imaginary parts makes solution for two instead of four modes sufficient.

Whenever possible the notation of the previous section is retained; some important exceptions are the angle ϕ which is replaced by PH, and the frequency ω which is replaced by OM. In general, quantities which pertain to the towed body start with a *B*. Thus m_s, the body added mass in surge, is *BMS*. The cable is of thickness *CH* and is *N*-1 segments long; each segment is of length *DS*. Its mass per unit length μ is replaced by *ULM* and its added mass parameter ν by *AMP*. Its weight per unit length in water is *WUL* and the density of water is *DN*. Further explanation of the program is provided by comment cards in the listing.

Each program includes four subroutines. *ATA* prevents the inverse-tangent function from dividing by a prohibitively small number if an angle is very close to $\pm \pi/2$. KUTMER performs integrations with a fourth-order Kutta-Merson method. This is a generalization of the Runge-Kutta method that provides an automatic reduction in integration-step size when an error criterion is not met. DAUX and DARN provide integrands for KUTMER; DAUX for ϕ_0, T_0, x_0, and y_0 and DARN for U_1, V_1, ϕ_1, and T_1. KUTMER is not used for x_1 and y_1; they are calculated by a simple first-order integration within the main program.

In the program OMWAY, *J* is an integer which increases from *J* = 1 at the towed body (s=0) to *J*=*N* at the towpoint (s=*L*). Successive data cards are used to specify;

1. Name of program, OMWAY.
2. Body lift *BL*, drag *BD*, and weight *BW*.
3. Magnitude and phase of surge and heave of towpoint, *ASM*, *DAS*, *AHM*, *DAH*,

and frequency *OM*.

4. Body mass and added masses in surge and heave, *BM*, *BMS*, and *BMH*.
5. *WUL*, *ULM*, *DS*, and *N*.
6. Tow speed *C*, *DN*, *CH*, cable drag coefficient *CD*, and *AMP*.
7. Constants for Equation [12] to give F_L/D_0.
8. Constants for Equation [13] to give F_G/D_0.

These data are written out. D_0 is computed and Equation [14] is used to compute λ, which is represented by AMC/ULM. Equations [33] and [34] are used to find T_0 and ϕ_0 at J=1 and U_0, V_0, ϕ_0, T_0, x_0, and y_0 are computed by using Equations [12], [13], [19], [20], [31], [32], [55], and [57] and are written out as functions of *J*. Values are assigned at J=1 as U_{1A} real and V_{1A} zero, ϕ_{1A} and T_{1A} are found from Equations [39] and [40], and the A-mode solution is computed by integrating Equations [35]–[38] on $1 < J \leq N$. Then U_{1B} is set zero and V_{1B} is set real at J=1 and the *B*-mode solution is found in the same way. Next c_{s1} and c_{h1} are computed and the required values of U_1 and V_1 at J=*N* are found from Equations [41] and [42] and C_A and C_B are found from Equations [49] and [50]. U_1, V_1, ϕ_1 and T_1 are found along the cable from Equations [45]–[48] and their magnitudes and phases are computed and written out. Finally Equations [56] and [58] are used to find x_1 and y_1 and their magnitudes and phases are computed and written out.

OMFLO uses the nomenclature of OMWAY wherever possible. *J* is an integer which decreases from J=-1 at the towed body ($s=0$) to $J=-N$ at the towpoint ($s=-L$). For performing integrations down the cable, which is done when ϕ_0, T_0, x_0, y_0, x_1, and y_1 are computed, an integer *K* is used which is defined by $K=-J$ and thus increases from $K=1$ at the towed body to $K=N$ at the towpoint. Since these integrations are made in the direction of decreasing *s*, the integrands computed in DAUX are negatives of those in OMWAY. U_1, V_1, ϕ_1, and T_1 are computed by integration in the direction of increasing *s* so an integer *I* is defined by $I=N+1-K$ which increases from $I=1$ at the towpoint to $I=N$ at the towed body. The first three data cards differ from those of OMWAY and specify:

1. Name of program, OMFLO.
2. Steady-state body lift and drag *BLO* and *BDO*, and body weight *BW*.
3. Magnitude and phase of oscillating lift and drag forces on the body, *BLM*, *DBL*, *DBM*, *DBD*, and frequency *OM*.

These data are written out. D_0 is computed and Equation [14] is used to compute λ, which is represented by AMC/ULM. Equations [33] and [34] are used to find

T_0 and ϕ_0 at $K=1$ and U_0, V_0, ϕ_0, T_0, x_0, and y_0 are computed by using Equations [12], [13], [19], [20], [31], [32], [55], and [57] and are written out as functions of J. ϕ_0 and T_0 are relisted as functions of I, ϕ_{IA} is set real and T_{IA}, U_{IA}, and V_{IA} are set equal to zero at $I=1$, and the A-mode solution is computed by integrating Equations [35]–[38] on $1 < I \leq N$. Then T_{IB} is set real and ϕ_{IB}, U_{IB}, and V_{IB} are are set equal to zero at $I=1$ and the B-mode solution is computed in the same way. Next ϕ_0, T_0, U_{IA}, V_{IA}, ϕ_{IA}, T_{IA}, U_{IB}, V_{IB}, ϕ_{IB}, and T_{IB} are re-listed as functions of K instead of I. The required values of P and Q at $K=1$ are found from Equations [39] and [40] and C_A and C_B are found from Equations [51] and [52]. U_I, V_I, ϕ_I, and T_I are found along the cable from Equations [45]–[48] and their magnitudes and phases are computed and written out. Finally Equations [56] and [58] are used to find x_I and y_I, and their magnitudes and phases are computed and written out.

Representative times for a run involving a cable nine integration steps long (of ten feet each) are 28 seconds compilation time and 14 seconds computation time on a CDC 6700 computer.

Certain modifications to the programs are easily made. For example, the programs listed require angles in radians and any consistent mass-length-time units for dimensional quantities. They have been adapted to treat, as input and printout quantities, angles in degrees, masses in pounds, lengths in feet, and speeds in knots. They have also been adapted to examine behavior over a range of frequencies, by repeating the unsteady part of the program in a DO loop, with lowest, highest, and incremental frequencies specified on an additional data card.

A computer experiment was performed on the Quadrant I program, OMWAY, by examining the frequency range 0.01 to 0.80 hertz in steps of 0.01 hertz for cable lengths of 200, 400, 600, 800, 1000, and 1200 feet at tow speeds of 6, 10, and 14 knots. As criteria for successful performance it was required that the perturbation quantities must remain below the steady-state quantities. It was found that V_M, the magnitude of the tangential velocity, at (or very near) the towpoint was the quantity which failed at the lowest frequency. The only cable length where its behavior was satisfactory throughout the frequency and speed ranges was 200 feet; all the longer ones failed but showed better behavior with increasing speed. The 600-foot cable failed at 0.26, 0.34, and 0.48 hertz at 6, 10, and 14 knots while the 1200-foot cable failed at 0.13, 0.20, and 0.30 hertz at the same speeds. Values for the 800-and 1000-foot cables lie in between these and exhibit the same monotonic relation to tow

speed. The 400-foot cable failed at 0.49 and 0.57 hertz at 6 and 10 knots and behaved satisfactorily at 14 knots.

The next most sensitive quantity was the perturbation tension at or very near the ship. It behaved well throughout the frequency and speed ranges for cable lengths of 800 feet and less. For the 1200 foot cable it failed at 0.46, 0.46, and 0.61 hertz at 6, 10, and 14 knots.

The qualitative conclusions are that the small-perturbation frequency-domain analysis is valid for short cables undergcing low-frequency oscilations and that increasing towspeed can have a stabilizing effect. Apparently as the forcing frequency approached 0.80 hertz, a resonant region was being approached; frequencies high enough to be past such a region, because the inertia of the cable and body prevent excitation of their motion, might be beyond the range of practical interest.

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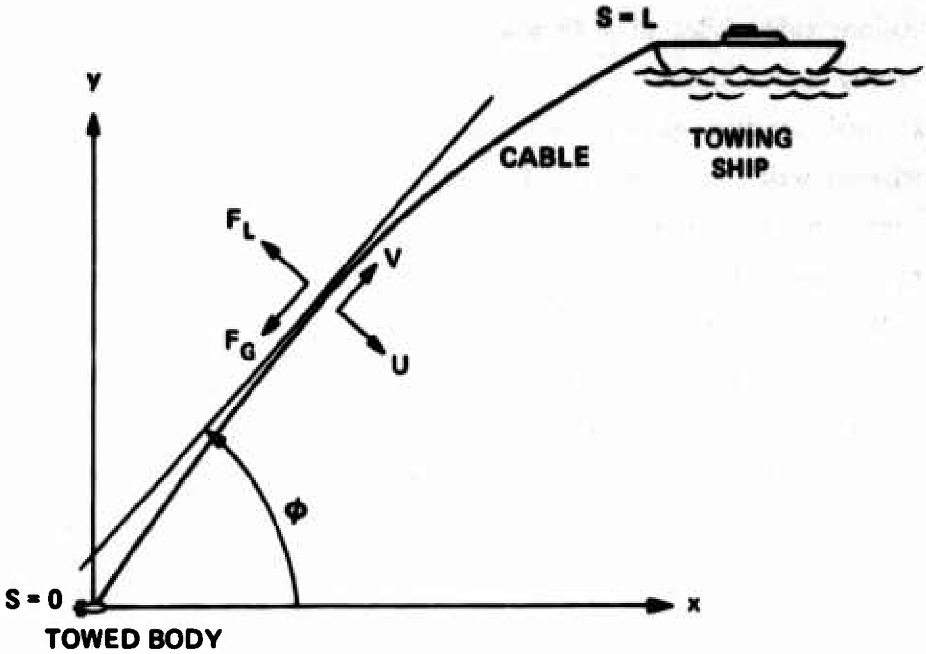

Figure 1 – Quadrant I Towing Configuration

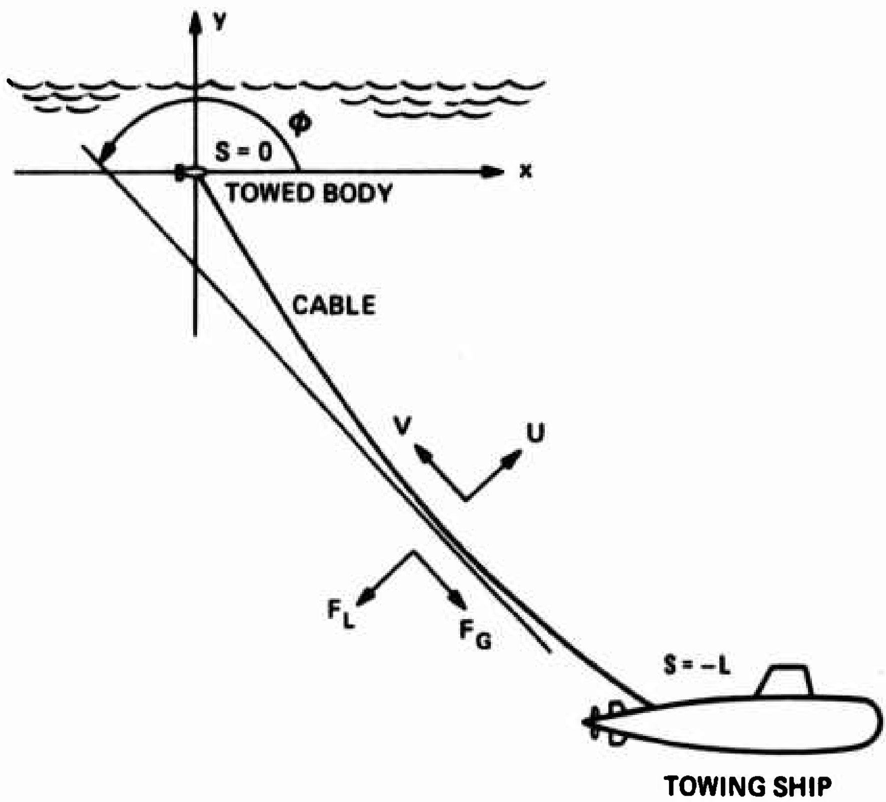

Figure 2 – Quadrant II Towing Configuration

APPENDIX A
DERIVATION OF EQUATIONS OF MOTION

Equations [1]–[4], the equations of cable motion, can be derived by considering a small element of the cable shown in Figure 1 or Figure 2. If x and y represent the coordinates of a point on the cable then x and y are functions of s and t which satisfy

$$\frac{\partial x}{\partial s} = \cos\phi \qquad [A1]$$

$$\frac{\partial y}{\partial s} = \sin\phi \qquad [A2]$$

$$\frac{\partial x}{\partial t}\sin\phi - \frac{\partial y}{\partial t}\cos\phi = U \qquad [A3]$$

and

$$\frac{\partial x}{\partial t}\cos\phi + \frac{\partial y}{\partial t}\sin\phi = V \qquad [A4]$$

Take the derivative of Equation [A3] with respect to s, interchange the order of the s- and t-differentiations of x and y, and substitute from Equations [A1] and [A2] to obtain

$$\frac{\partial U}{\partial s} = \frac{\partial}{\partial t}(\cos\phi)\sin\phi + \frac{\partial x}{\partial t}\cos\phi\frac{\partial \phi}{\partial s} - \frac{\partial}{\partial t}(\sin\phi)\cos\phi + \frac{\partial y}{\partial t}\sin\phi\frac{\partial \phi}{\partial s}$$

which becomes Equation [1] after substitution of Equation [A4]. In the same way, take the derivative of Equation [A4] with respect to s, interchange the order of differentiations, and substitute to obtain

$$\frac{\partial V}{\partial s} = \frac{\partial}{\partial t}(\cos\phi)\cos\phi - \frac{\partial x}{\partial t}\sin\phi\frac{\partial \phi}{\partial s} + \frac{\partial}{\partial t}(\sin\phi)\sin\phi + \frac{\partial y}{\partial t}\cos\phi\frac{\partial \phi}{\partial s}$$

This, with the substitution of Equation [A3], is Equation [2].

The apparent-momentum vector of a segment of length δs has components $\mu\delta s(1+\lambda)U$, $\mu\delta sV$, 0 in a right-handed orthogonal coordinate system with axes in the directions of U, V, and upwards from the paper of Figure 1 or Figure 2. Here μ is the mass of the cable per unit length and $\mu\lambda$ is the added mass in water for

motion in the direction of U. The angular velocity has components $0, 0, \partial\phi/\partial t$ in the fixed coordinate system defined by the x and y axes and the direction up out of the paper in either figure. Therefore the rate of change of apparent momentum observed from the fixed coordinate system in either figure has components along the directions of U and V equal to

$$\mu\, \delta s(1+\lambda)\frac{\partial U}{\partial t} - \mu \delta s V \frac{\partial \phi}{\partial t} \qquad \text{and} \qquad \mu \delta s \frac{\partial V}{\partial t} + \mu \delta s(1+\lambda)\, U \frac{\partial \phi}{\partial t}$$

Therefore the dynamic equations of motion of the cable segment are

$$\mu \delta s \left[(1+\lambda)\frac{\partial U}{\partial t} - V\frac{\partial \phi}{\partial t}\right] = \text{resultant force on } \delta s \text{ in the direction of } U \quad \text{[A5]}$$

and

$$\mu \delta s \left[\frac{\partial V}{\partial t} + (1+\lambda)\, U\frac{\partial \phi}{\partial t}\right] = \text{resultant force on } \delta s \text{ in the direction of } V \quad \text{[A6]}$$

The right-hand sides of Equations [A5] and [A6] are easily computed and the results are, after division by δs, Equations [3] and [4].

APPENDIX B

LISTING AND SAMPLE OUTPUT OF OMWAY

```
      PROGRAM OMWAY(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION PHO(400), TO(400), UO(400), VO(400), PHRA(400),
     + PHRB(400), PHIA(400), PHIB(400), TRA(400), TRB(400), TIA(400),
     + TIB(400), URA(400), URB(400), UIA(400), UIB(400), VRA(400),
     + VRB(400), VIA(400), VIB(400),PHR(400), PHI(400), TR(400),
     + TI(400), UR(400), UI(400), VR(400), VI(400), FL(400), FG(400),
     + DFL(400), DFG(400), T(8)
      COMMON DS,WUL,D,ALAM0,ALAM1,ALAM2,BLAM1,
     1BLAM2,AGAM0,AGAM1,AGAM2,BGAM1,BGAM2,
     2OM,ULM,APP,C,PHO,DFL,DFG,J,TO,PHR,PHI,FL,FG
      EXTERNAL DAUX,DARN
      READ (5,11) TITLE,
     + BL , BD , BW,
     + ASM, DAS, AHM, DAH, OM,
     + BM, BMS, BMH,
     + WUL, ULM, DS, N,
X    + C, DN, CH, CD, AMP,
     + ALAM0, ALAM1, ALAM2, BLAM1, BLAM2,
     + AGAM0, AGAM1, AGAM2, BGAM1, BGAM2
C   WRITE TITLE AND INPUT DATA
      WRITE (6,21) TITLE
      WRITE (6,22) BL , BD , BW
      WRITE (6,23) ASM, DAS, AHM, DAH, OM
      WRITE (6,24) BM, BMS, BMH
      WRITE (6,25) WUL, ULM, DS, N
      WRITE (6,26) C, DN, CH, CD, AMP
      WRITE (6,27) ALAM0, ALAM1, ALAM2, BLAM1, BLAM2
      WRITE (6,28) AGAM0, AGAM1, AGAM2, BGAM1, BGAM2
C   INITIALIZE
      D= 0.5*DN*C*C*CH*CD
      AMC  =  3.141593*CH*CH*DN*AMP/4.0
      APP  =   1.0 + AMC/ULM
      CXO = 0.0
      CYO = 0.0
      CXR = 0.0
      CXI = 0.0
      CYR = 0.0
      CYI = 0.0
      JA  =  N-1
C   SOLVE STEADY-STATE PROBLEM
      ERR=.00001
      ERA=.001
      CALL ATA(BD,YK)
      PHOB = ATAN2(BW-BL,YK)
      TOB = SQRT(BD*BD+(BW-BL)*(BW-BL))
      WRITE (6,31)
      WRITE (6,32)
      PHO(1) = PHOB
      TO(1) = TOB
C   MULTIPLY TO FIND VELOCITY COMPONENTS
      DO 101  J = 1,N
      UO(J) = C*SIN(PHO(J))
      VO(J) = C*COS(PHO(J))
      WRITE (6,33) PHO(J), TO(J), UO(J), VO(J), CXO, CYO, J
      IF (J.EQ.N)  GO TO 101
```

```
C     INTEGRATE TO FIND TENSION, ANGLE, AND CABLE CONFIGURATION
         T(1)=T0(J)
         T(2)=PH0(J)
         T(3)  =  CX0
         T(4)  =  CY0
         FIRST=.0
         ALS=FLOAT(J)*DS
         CALL KUTMER(4,ALS,T,ERR,DS,FIRST,HCX,ERA,DAUX)
         T0(J+1)=T(1)
         PH0(J+1)=T(2)
         CX0  =  T(3)
         CY0  =  T(4)
101      CONTINUE
C     SOLVE DYNAMIC PROBLEM
         WRITE (6,34)
         WRITE (6,35)
C     COMPUTE VISCOUS-FORCE TERMS ALONG CABLE
         DO 201  J = 1, JA
         FL(J) = D*(+ALAM0+ALAM1*COS(PH0(J))+ALAM2*COS(2.0*PH0(J))+
      + BLAM1*SIN(PH0(J))+BLAM2*SIN(2.0*PH0(J)))
         FG(J) = D*( AGAM0+AGAM1*COS(PH0(J))+AGAM2*COS(2.0*PH0(J))
      + +BGAM1*SIN(PH0(J))+BGAM2*SIN(2.0*PH0(J)))
         DFL(J) = D*(-ALAM1*SIN(PH0(J))-2.0*ALAM2*SIN(2.0*PH0(J))
      + +BLAM1*COS(PH0(J))+2.0*BLAM2*COS(2.0*PH0(J)))
         DFG(J) = D*(-AGAM1*SIN(PH0(J))-2.0*AGAM2*SIN(2.0*PH0(J))
      + +BGAM1*COS(PH0(J))+2.0*BGAM2*COS(2.0*PH0(J)))
201      CONTINUE
C     COMPUTE MODAL SOLUTIONS
C     COMPUTE  A -MODE SOLUTION
C     ASSIGN VALUES TO VELOCITY COMPONENTS AT BOTTOM
         URA(1) = 0.000001
         UIA(1) = 0.0
         VRA(1) = 0.0
         VIA(1) = 0.0
C     COMPUTE CORRESPONDING VALUES OF ANGLE AND TENSION AT BOTTOM
         PHRA(1) =     OM*(-UIA(1)*(BM+BMS*SIN(PH0(1))*SIN(PH0(1))+
      + BMH*COS(PH0(1))*COS(PH0(1)))+VIA(1)*(BMH-BMS)*SIN(PH0(1))*COS(
      + PH0(1)))/T0(1)
         PHIA(1) =    -OM*(-URA(1)*(BM+BMS*SIN(PH0(1))*SIN(PH0(1))+
      + BMH*COS(PH0(1))*COS(PH0(1)))+VRA(1)*(BMH-BMS)*SIN(PH0(1))*COS(
      + PH0(1)))/T0(1)
         TRA(1) =     OM*(-UIA(1)*(BMH-BMS)*SIN(PH0(1))*COS(PH0(1))
      + +VIA(1)*(BM+BMS*COS(PH0(1))*COS(PH0(1))+
      + BMH*SIN(PH0(1))*SIN(PH0(1))))
         TIA(1) =    -OM*(-URA(1)*(BMH-BMS)*SIN(PH0(1))*COS(PH0(1))
      + +VRA(1)*(BM+BMS*COS(PH0(1))*COS(PH0(1))+
      + BMH*SIN(PH0(1))*SIN(PH0(1))))
C     FIND VELOCITY COMPONENTS, ANGLE, AND TENSION BY INTEGRATING UP
C     THE CABLE
         DO 202  J = 1, JA
         T(1)=PHRA(J)
         T(2)=PHIA(J)
         T(3)=TRA(J)
         T(4)=TIA(J)
         T(5)=URA(J)
```

```
      T(6)=UIA(J)
      T(7)=VRA(J)
      T(8)=VIA(J)
      FIRST=.0
      ALS=FLOAT(J)*DS
      CALL KUTMER(8,ALS,T,ERR,DS,FIRST,HCX,ERA,DARN)
      PHRA(J+1)=T(1)
      PHIA(J+1)=T(2)
      TRA(J+1)=T(3)
      TIA(J+1)=T(4)
      URA(J+1)=T(5)
      UIA(J+1)=T(6)
      VRA(J+1)=T(7)
      VIA(J+1)=T(8)
  202 CONTINUE
C   COMPUTE  B -MODE SOLUTION
C   ASSIGN VALUES TO VELOCITY COMPONENTS AT BOTTOM
      URB(1) = 0.0
      UIB(1) = 0.0
      VRB(1) = 0.000001
      VIB(1) = 0.0
C   COMPUTE CORRESPONDING VALUES OF ANGLE AND TENSION AT BOTTOM
      PHRB(1) =    OM*(-UIB(1)*(BM+BMS*SIN(PHO(1))*SIN(PHO(1))+
     + BMH*COS(PHO(1))*COS(PHO(1)))+VIB(1)*(BMH-BMS)*SIN(PHO(1))*COS(
     + PHO(1)))/TO(1)
      PHIB(1) =   -OM*(-URB(1)*(BM+BMS*SIN(PHO(1))*SIN(PHO(1))+
     + BMH*COS(PHO(1))*COS(PHO(1)))+VRB(1)*(BMH-BMS)*SIN(PHO(1))*COS(
     + PHO(1)))/TO(1)
      TRB(1) =    OM*(-UIB(1)*(BMH-BMS)*SIN(PHO(1))*COS(PHO(1))
     + +VIB(1)*(HM+BMS*COS(PHO(1))*COS(PHO(1))+
     + BMH*SIN(PHO(1))*SIN(PHO(1))))
      TIB(1) =   -OM*(-URB(1)*(BMH-BMS)*SIN(PHO(1))*COS(PHO(1))
     + +VRB(1)*(BM+HMS*COS(PHO(1))*COS(PHO(1))+
     + BMH*SIN(PHO(1))*SIN(PHO(1))))
C   FIND VELOCITY COMPONENTS, ANGLE, AND TENSION BY INTEGRATING UP
C   THE CABLE
      DO 203  J = 1, JA
      T(1)=PHRB(J)
      T(2)=PHIB(J)
      T(3)=TRB(J)
      T(4)=TIB(J)
      T(5)=URB(J)
      T(6)=UIB(J)
      T(7)=VRB(J)
      T(8)=VIB(J)
      FIRST=.0
      ALS=FLOAT(J)*DS
      CALL KUTMER(8,ALS,T,ERR,DS,FIRST,HCX,ERA,DARN)
      PHRB(J+1)=T(1)
      PHIB(J+1)=T(2)
      TRB(J+1)=T(3)
      TIB(J+1)=T(4)
      URB(J+1)=T(5)
      UIB(J+1)=T(6)
      VRB(J+1)=T(7)
```

```
      VIB(J+1)=T(8)
203   CONTINUE
C     SUPERIMPOSE MODAL SOLUTIONS
C     COMPUTE VELOCITY COMPONENTS FORCED AT TOP
      CSR=OM*ASM*SIN(DAS)
      CSI=-OM*ASM*COS(DAS)
      CHR=OM*AHM*SIN(DAH)
      CHI=-OM*AHM*COS(DAH)
      UR(N)   =  CSR*SIN(PHO(N)) - CHR*COS(PHO(N))
      UI(N)   =  CSI*SIN(PHO(N)) - CHI*COS(PHO(N))
      VR(N)   =  CSR*COS(PHO(N)) + CHR*SIN(PHO(N))
      VI(N)   =  CSI*COS(PHO(N)) + CHI*SIN(PHO(N))
C     COMPUTE CONSTANTS NEEDED TO SUPERIMPOSE MODAL SOLUTIONS
      DENOMR   =  URA(N)*VRB(N) - UIA(N)*VIB(N) - URB(N)*VRA(N) +
     + UIB(N)*VIA(N)
      DENOMI   =  UIA(N)*VRB(N) + URA(N)*VIB(N) - UIB(N)*VRA(N) -
     + URB(N)*VIA(N)
      ANUMR    =  UR (N)*VRB(N) - UI  (N)*VIB(N) - URB(N)*VR  (N) +
     + UIB(N)*VI  (N)
      ANUMI    =  UI  (N)*VRB(N) + UR  (N)*VIB(N) - UIB(N)*VR  (N) -
     + URB(N)*VI  (N)
      BNUMR    =  URA(N)*VR  (N) - UIA(N)*VI  (N) - UR  (N)*VRA(N) +
     + UI  (N)*VIA(N)
      BNUMI    =  UIA(N)*VR  (N) + URA(N)*VI  (N) - UI  (N)*VRA(N) -
     + UR  (N)*VIA(N)
      DENOM2   =  DENOMR*DENOMR + DENOMI*DENOMI
      CAR   =   (ANUMR*DENOMR + ANUMI*DENOMI)/DENOM2
      CAI   =  (-ANUMR*DENOMI + ANUMI*DENOMR)/DENOM2
      CBR   =   (BNUMR*DENOMR + BNUMI*DENOMI)/DENOM2
      CBI   =  (-BNUMR*DENOMI + BNUMI*DENOMR)/DENOM2
C     COMPUTE VELOCITY COMPONENTS, ANGLE, AND TENSION ON CABLE
      DO 204   J = 1,N
      UR(J)   =  CAR*URA(J) - CAI*UIA(J) + CBR*URB(J) - CBI*UIB(J)
      UI(J)   =  CAI*URA(J) + CAR*UIA(J) + CBI*URB(J) + CBR*UIB(J)
      VR(J)   =  CAR*VRA(J) - CAI*VIA(J) + CBR*VRB(J) - CBI*VIB(J)
      VI(J)   =  CAI*VRA(J) + CAR*VIA(J) + CBI*VRB(J) + CBI*VIB(J)
      PHR(J)   =  CAR*PHRA(J) - CAI*PHIA(J) + CBR*PHRB(J) - CBI*PHIB(J)
      PHI(J)   =  CAI*PHRA(J) + CAR*PHIA(J) + CBI*PHRB(J) + CBR*PHIB(J)
      TR(J)   =  CAR*TRA(J) - CAI*TIA(J) + CBR*TRB(J) - CBI*TIB(J)
      TI(J)   =  CAI*TRA(J) + CAR*TIA(J) + CBI*TRB(J) + CBR*TIB(J)
C     COMPUTE PHYSICAL VELOCITY COMPONENTS ON CABLE
      URP   =  C*PHR(J)*COS(PHO(J)) + UR(J)
      UIP   =  C*PHI(J)*COS(PHO(J)) + UI(J)
      VRP   =  - C*PHR(J)*SIN(PHO(J)) + VR(J)
      VIP   =  - C*PHI(J)*SIN(PHO(J)) + VI(J)
C     COMPUTE MAGNITUDE AND PHASE OF VELOCITY COMPONENTS, ANGLE, AND
C     TENSION ON CABLE
      UM   =  SQRT(URP*URP + UIP*UIP)
      CALL ATA(URP, YK)
      DU   =  ATAN2(UIP, YK)
      VM   =  SQRT(VRP*VRP + VIP*VIP)
      CALL ATA(VRP, YK)
      DV   =  ATAN2(VIP, YK)
      PHM   =  SQRT(PHR(J)*PHR(J) + PHI(J)*PHI(J))
      CALL ATA(PHR(J), YK)
```

```
          DPH  =  ATAN2(PHI(J), YK)
          TM  =  SQRT(TR(J)*TR(J) + TI(J)*TI(J))
          CALL ATA(TR(J), YK)
          DT  =  ATAN2(TI(J), YK)
          WRITE (6,36) PHM, DPH, TM, DT, UM, DU, VM, DV, J
    204   CONTINUE
          WRITE (6,37)
C   COMPUTE MAGNITUDE AND PHASE OF DYNAMIC CABLE CONFIGURATION
          DO 205  J = 1, N
          CXM = SQRT(CXR*CXR+CXI*CXI)
          CALL ATA(CXR, YK)
          DCX  =  ATAN2(CXI, YK)
          CYM = SQRT(CYR*CYR+CYI*CYI)
          CALL ATA(CYR, YK)
          DCY  =  ATAN2(CYI, YK)
          WRITE (6,38) CXM, DCX, CYM, DCY, J
          IF (J.EQ. N) GO TO 205
          CXR  =  - PHR(J)*SIN(PHO(J))*DS
          CXI  =  - PHI(J)*SIN(PHO(J))*DS
          CYR  =    PHR(J)*COS(PHO(J))*DS
          CYI  =    PHI(J)*COS(PHO(J))*DS
    205   CONTINUE
    11    FORMAT ( 10X,  A6    / 10X, 3F10.5/ 10X, 5F10.5/ 10X, 3F10.5/
         +  10X, 3F10.5, I10/10X,5F10.5 / 10X, 5F10.5/ 10X, 5F10.5)
    21    FORMAT (1H1, 10X,       57HSURFACE SHIP IN A SEAWAY TOWING A DEEPLY
         + SUBMERGED WEIGHT/ 1H-, 10X, A6///)
    22    FORMAT ( 1X, 8HBL     = , F10.5, 4X, 8HBD     = , F10.5, 4X,
         + 8HBW     = , F10.5 //)
     23   FORMAT ( 1X, 8HASM    = , F10.5, 4X, 8HDAS    = , F10.5, 4X,
         + 8HAHM    = , F10.5, 4X, 8HDAH    = , F10.5, 4X, 8HOM     = ,
         + F10.5 //)
    24    FORMAT ( 1X, 8HBM     = , F10.5, 4X, 8HBMS    = , F10.5, 4X,
         + 8HBMH    = , F10.5 //)
    25    FORMAT ( 1X, 8HWUL    = , F10.5, 4X, 8HULM    = , F10.5, 4X,
         + 8HDS     = , F10.5, 4X, 8HN      = , I10 //)
    26    FORMAT ( 1X, 8HC      = , F10.5, 4X, 8HON     = , F10.5, 4X,
         +8HCH     = ,F10.5, 4X, 8HCD     = ,F10.5, 4X, 8HAMP    = ,F10.5//)
    27    FORMAT ( 1X, 8HALAMO = ,F10.5, 4X, 8HALAM1 = , F10.5,4X,
         + 8HALAM2 = , F10.5, 4X, 8HBLAM1 = , F10.5, 4X, 8HBLAM2 = ,
         + F10.5 //)
    28    FORMAT ( 1X, 8HAGAMO = , F10.5, 4X, 8HAGAM1 = , F10.5, 4X,
         + 8HAGAM2 = , F10.5, 4X, 8HBGAM1 = , F10.5, 4X, 8HBGAM2 = ,
         + F10.5 //)
    31    FORMAT ( 22H1STEADY-STATE SOLUTION)
    32    FORMAT ( 1H0, 1X, 6HPHO(J), 10X, 6HTO(J) , 10X, 6HUO(J) , 10X,
         + 6HVO(J) , 10X, 6HCXO(J), 10X, 6HCYO(J), 10X, 1HJ  //)
    33    FORMAT ( 2X, F9.6, 7X, F13.6, 3X, 2( F11.6, 5X), 2(F13.6, 3X), I4)
    34    FORMAT ( 17H1DYNAMIC SOLUTION)
    35    FORMAT ( 1H0, 1X, 6HPHM(J), 9X, 6HDPH(J), 9X, 6HTM(J) , 9X,
         + 6HDT(J) , 9X, 6HUM(J) , 9X, 6HDU(J) , 9X, 6HVM(J) , 9X, 6HDV(J) ,
         + 9X, 1HJ //)
    36    FORMAT (      2X, 2( F9.6, 6X), F12.6, 3X, F9.6, 6X, 2(F10.6, 5X,
         + F9.6, 6X), I4)
    37    FORMAT ( 1H1, 1X, 6HCXM(J), 10X, 6HDCX(J), 10X, 6HCYM(J), 10X,
         + 6HDCY(J), 10X, 1HJ //)
    38    FORMAT ( 2X, 2( F12.6, 4X, F9.6, 7X), I4)
          STOP
          END
```

```
      SUBROUTINE  ATA(X,YK)
C   THIS SUBROUTINE PREVENTS  ATAN2  FROM DIVIDING BY ZERO WHEN
C   COMPUTING THE ARCTANGENTS OF  PI/2  AND  3*PI/2
      ER = 0.00001
      IF (ABS(X).GT.ER) GO TO 10
      YK = ER
      RETURN
10    YK = X
      RETURN
      END
```

```
      SUBROUTINE KUTMER(N,T,Y0,EPS,H,FIRST,HCX,A,DAUX)
C   THIS SUBROUTINE PERFORMS FOURTH-ORDER KUTTA-MERSON INTEGRATIONS
      DIMENSION Y0(10),Y1(10),Y2(10),F0(10),F1(10),F2(10)
      IF(FIRST)20,10,20
   10 HC=H                                                              KUTM0050
      IPLOC=1                                                           KUTM0060
      FIRST=1.                                                          KUTM0070
   20 LOC=0                                                             KUTM0080
        HCX=HC                                                          KUTM0090
   30 CALL DAUX(T,Y0,F0)                                                KUTM0100
   39 DO 40 I=1,N                                                       KUTM0110
   40 Y1(I)=Y0(I)+(HC/3.)*F0(I)                                         KUTM0120
      CALL DAUX(T+HC/3.,Y1,F1)                                          KUTM0130
      DO 50 I=1,N                                                       KUTM0140
   50 Y1(I)=Y0(I)+(HC/6.)*F0(I)+(HC/6.)*F1(I)                           KUTM0150
      CALL DAUX(T+HC/3.,Y1,F1)                                          KUTM0160
      DO 60 I=1,N                                                       KUTM0170
   60 Y1(I)=Y0(I)+HC/8.*F0(I)+.375*HC*F1(I)                             KUTM0180
      CALL DAUX(T+HC/2.,Y1,F2)                                          KUTM0190
      DO 70 I=1,N                                                       KUTM0200
   70 Y1(I)=Y0(I)+HC/2.*F0(I)-1.5*HC*F1(I)+2.*HC*F2(I)                  KUTM0210
      CALL DAUX(T+HC,Y1,F1)                                             KUTM0220
      DO 80 I=1,N                                                       KUTM0230
   80 Y2(I)=Y0(I)+HC/6.*F0(I)+.66666667*HC*F2(I)+(HC/6.)*F1(I)          KUTM0240
      INC=0                                                             KUTM0250
      DO 110 I=1,N                                                      KUTM0260
      ZZZ=ABS(Y1(I))-A                                                  KUTM0270
        IF(ZZZ) 85,87,87                                                KUTM0280
   85 ERROR=ABS(.2*(Y1(I)-Y2(I)))                                       KUTM0290
        IF(ERROR-A)100,100,90                                           KUTM0300
   87 ERROR=ABS(.2-.2*Y2(I)/Y1(I))                                      KUTM0310
      IF(ERROR-EPS)100,100,90                                           KUTM0320
   90 X=128.*ABS(HC)-ABS(H)                                             KUTM0330
        IF(X) 91,95,95                                                  KUTM0340
   91 WRITE(6,92) T,ERROR                                               KUTM0350
   92 FORMAT(21H RELATIVE ERROR AT T= 1P1E12.3,3HIS F10.6     )         KUTM0360
        FIRST = 2.                                                      KUTM0370
        RETURN                                                          KUTM0380
   95 HC=HC/2.                                                          KUTM0390
      IPLOC=2 *IPLOC                                                    KUTM0400
      LOC=2 *LOC                                                        KUTM0410
        HCX=HC                                                          KUTM0420
      GO TO 30                                                          KUTM0430
  100 IF(ERROR*64.-EPS)110,110,101                                      KUTM0440
  101 INC=1                                                             KUTM0450
  110 CONTINUE                                                          KUTM0460
  111 T=T+HC                                                            KUTM0470
      DO 112 I=1,N                                                      KUTM0480
  112 Y0(I)=Y2(I)                                                       KUTM0490
      LOC=LOC+1                                                         KUTM0500
       IF(LOC-IPLOC)120,210,210                                         KUTM0510
  120 IF(INC)210,130,210                                                KUTM0520
  130 IF(LOC-(LOC/2)*2)210,140,210                                      KUTM0530
  140 IF(IPLOC-1)210,210,200                                            KUTM0540
  200 HC=2.*HC                                                          KUTM0550
```

```
      LOC=LOC/2                                                         KUTM0560
      IPLOC=IPLOC/2                                                     KUTM0570
  210 IF(IPLOC-LOC)30,220,30                                            KUTM0580
  220 RETURN                                                            KUTM0590
      END                                                               KUTM0600
```

```
      SUBROUTINE DAUX(Z,TX,F)
C     THIS SUBROUTINE PROVIDES THE INTEGRANDS FOR  KUTMER  WHEN IT
C     COMPUTES THE STEADY-STATE TENSION, ANGLE, AND CABLE CONFIGURATION
      DIMENSION  TX(8), F(8),PHO(400),DFL(400),DFG(400),TO(400),
     + PHR(200), PHI(200)
      COMMON DS,WUL,D,ALAM0,ALAM1,ALAM2,BLAM1,
     1BLAM2,AGAM0,AGAM1,AGAM2,BGAM1,BGAM2,
     2OM,ULM,APP,C,PHO,DFL,DFG,J,TO,PHR,PHI
      TX1=TX(1)
      TX2=TX(2)
      C1=COS(TX2)
      S1=SIN(TX2)
      C2=COS(2.*TX2)
      S2=SIN(2.*TX2)
      FL=D*(ALAM0+ALAM1*C1+ALAM2*C2+BLAM1*S1+BLAM2*S2)
      FG= D*(AGAM0+AGAM1*C1+AGAM2*C2+BGAM1*S1+BGAM2*S2)
      IF(TX1.EQ.0.) GO TO 50
      F(2)= (WUL*C1-FL)/TX1
      F(3)  =  C1
      F(4)  =  S1
    7 F(1)= (WUL*S1+FG)
      RETURN
   50 F(2)=0.0
      GO TO 7
      END
```

```
      SUBROUTINE DARN(Z,T,F)
C     THIS SUBROUTINE PROVIDES THE INTEGRANDS FOR  KUTMER  WHEN IT
C     COMPUTES THE DYNAMIC ANGLE, TENSION, AND VELOCITY COMPONENTS
      DIMENSION  T(8), F(8),PHO(400),DFL(400),DFG(400),TO(400),
     + FL(400),FG(400),PHR(400),PHI(400)
      COMMON DS,WUL,D,ALAM0,ALAM1,ALAM2,BLAM1,
     1BLAM2,AGAM0,AGAM1,AGAM2,BGAM1,BGAM2,
     2OM,ULM,APP,C,PHO,DFL,DFG,J,TO,PHR,PHI,FL,FG
      C1=COS(PHO(J))
      S1=SIN(PHO(J))
      IF(TO(J).EQ.0.0) GO TO 50
      F(1)=(-OM*ULM*(APP*T(6)+(APP-1.0)*C*T(2)*C1)-T(3)*(PHO(J+1)-PHO(J)
     +)/DS-(T(1)+(T(5)*C1-T(7)*S1)/C)*DFL(J)
     +                  -2.0*FL(J)*(T(5)*S1+T(7)*C1)/C-WUL*T(1)*S1)/TO(J)
      F(2)=( OM*ULM*(APP*T(5)+(APP-1.0)*C*T(1)*C1)-T(4)*(PHO(J+1)-PHO(J)
     +)/DS-(T(2)+(T(6)*C1-T(8)*S1)/C)*DFL(J)
     +                  -2.0*FL(J)*(T(6)*S1+T(8)*C1)/C-WUL*T(2)*S1)/TO(J)
    7 F(3)=( OM*ULM*(T(8)+(APP-1.0)*C*T(2)*S1)+(T(1)+(T(5)*C1-
     + T(7)*S1)/C)*DFG(J)+2.0*FG(J)*
     + (T(5)*S1+T(7)*C1)/C+WUL*T(1)*C1)
      F(4)=(-OM*ULM*(T(7)+(APP-1.0)*C*T(1)*S1)+(T(2)+(T(6)*C1-
     + T(8)*S1)/C)*DFG(J)+2.0*FG(J)*
     + (T(6)*S1+T(8)*C1)/C+WUL*T(2)*C1)
      F(5)=-OM*T(2)+T(7)*(PHO(J+1)-PHO(J))/DS
      F(6)= OM*T(1)+T(8)*(PHO(J+1)-PHO(J))/DS
      F(7)=-T(5)*(PHO(J+1)-PHO(J))/DS
      F(8)=-T(6)*(PHO(J+1)-PHO(J))/DS
      RETURN
   50 F(1)=0.0
      F(2)=0.0
      GO TO 7
      END
```

SURFACE SHIP IN A SEAWAY TOWING A DEEPLY SUBMERGED WEIGHT
OMWAY

BL	=	0.00000	BD	=	859.00000	BW	=	6040.00000						
ASM	=	0.00000	DAS	=	0.00000	AHM	=	1.00000	DAH	=	0.00000	OM	=	3.00000
BM	=	59.00000	BMS	=	1.55200	BMH	=	14.58000						
WUL	=	.68000	ULM	=	.02730	DS	=	10.00000	N	=	20			
C	=	23.60000	DN	=	1.99040	CH	=	.06250	CD	=	1.90000	AMP	=	1.00000
ALAMO	=	.47500	ALAM1	=	0.00000	ALAM2	=	-.47500	BLAM1	=	.05000	BLAM2	=	0.00000
AGAMO	=	0.00000	AGAM1	=	.05000	AGAM2	=	0.00000	BGAM1	=	0.00000	BGAM2	=	0.00000

STEADY-STATE SOLUTION

PHO(J)	TO(J)	UO(J)	VO(J)	CXO(J)	CYO(J)	J
1.429525	6100.777082	23.364892	3.322921	0.000000	0.000000	1
1.325929	6113.771030	22.896003	5.721281	1.921487	9.809099	2
1.227636	6129.825151	22.224023	7.940580	4.823133	19.374659	3
1.135983	6148.608529	21.403985	9.941299	8.619797	28.622110	4
1.051700	6169.768338	20.491126	11.707850	13.214786	37.500561	5
.974994	6192.959022	19.533680	13.243691	18.509051	45.981229	6
.905677	6217.861785	18.569499	14.564811	24.407890	54.053629	7
.843314	6244.194678	17.625688	15.693793	30.824960	61.721014	8
.787333	6271.715587	16.719984	16.655394	37.684075	68.996069	9
.737111	6300.220740	15.862777	17.473761	44.919480	75.897365	10
.692026	6329.540867	15.059120	18.170936	52.475242	82.446679	11
.651492	6359.536466	14.310420	18.766243	60.304203	88.667102	12
.614975	6390.093060	13.615734	19.276198	68.366795	94.581787	13
.581994	6421.116843	12.972702	19.714690	76.629882	100.213157	14
.552127	6452.530925	12.378189	20.093293	85.065711	105.582455	15
.525004	6484.272167	11.828711	20.421596	93.651000	110.709508	16
.500302	6516.288590	11.320703	20.707527	102.366170	115.612642	17
.477742	6548.537265	10.850686	20.957638	111.194699	120.308688	18
.457079	6580.982604	10.415361	21.177352	120.122592	124.813037	19
.438103	6613.594998	10.011657	21.371166	129.137953	129.139729	20

DYNAMIC SOLUTION

PHM(J)	DPH(J)	TM(J)	DT(J)	UM(J)	DU(J)	VM(J)	DV(J)	J
.008842	.837901	301.489251	-2.814459	.331768	-.704693	1.489530	-1.362528	1
.005794	1.576827	302.732956	-2.807222	.015479	1.880557	1.519121	-1.264096	2
.005234	2.339834	303.882394	-2.800017	.237110	2.793122	1.494022	-1.209241	3
.005682	2.839033	305.049831	-2.792912	.372358	3.009067	1.450318	-1.189389	4
.006049	3.106924	306.306609	-2.785766	.448404	-3.078312	1.406860	-1.193621	5
.006048	-3.060084	307.687693	-2.778396	.494144	-2.899734	1.364827	-1.209794	6
.005691	-3.061654	309.209469	-2.770663	.531310	-2.750316	1.312969	-1.227736	7
.005088	3.107297	310.884818	-2.762504	.571834	-2.646450	1.235025	-1.240898	8
.004406	2.865046	312.730100	-2.753937	.619215	-2.599972	1.116202	-1.245479	9
.003893	2.470387	314.764821	-2.745055	.672496	-2.614224	.947426	-1.237930	10
.003873	1.946461	317.006731	-2.736011	.729948	-2.686214	.727655	-1.210339	11
.004559	1.416073	319.464968	-2.727006	.791120	-2.810360	.465438	-1.133550	12
.005893	.972827	322.133194	-2.718269	.857743	-2.980820	.187316	-.792759	13
.007739	.611171	324.983988	-2.710038	.934251	3.091259	.172282	1.203138	14
.010034	.300525	327.965323	-2.702531	1.028245	2.845614	.432074	1.509243	15
.012783	.019702	330.999525	-2.695925	1.150712	2.573098	.651397	1.683628	16
.016029	-.243020	333.984878	-2.690330	1.315609	2.283401	.782201	1.735628	17
.019836	-.493996	336.799772	-2.685773	1.538719	1.987242	.788414	1.783612	18
.024286	-.736794	339.309145	-2.682178	1.836279	1.694194	.639274	1.858182	19
.029477	-.973510	341.372911	-2.679355	2.224183	1.410826	.319807	2.116448	20

CXM(J)	DCX(J)	CYM(J)	DCY(J)	J
0.000000	0.000000	0.000000	0.000000	1
.087537	-2.303692	.012449	.837901	2
.056209	-1.564765	.014046	1.576827	3
.049291	-.801759	.017611	2.339834	4
.051531	-.302560	.023934	2.839033	5
.052518	-.034669	.030007	3.106924	6
.050056	.081509	.033937	-3.060084	7
.044777	.079939	.035120	-3.061654	8
.037998	-.034296	.033833	3.107297	9
.031219	-.276547	.031098	2.865046	10
.026169	-.671205	.028826	2.470387	11
.024713	-1.195132	.029820	1.946461	12
.027646	-1.725520	.036254	1.416073	13
.033999	-2.168766	.048134	.972827	14
.042539	-2.530422	.064647	.611171	15
.052629	-2.841067	.085431	.300525	16
.064073	-3.121891	.110618	.019702	17
.076891	2.898573	.140647	-.243020	18
.091202	2.647597	.176153	-.493996	19
.107182	2.404798	.217931	-.736794	20

APPENDIX C

LISTING AND SAMPLE OUTPUT OF OMFLO

```
      PROGRAM OMFLO(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION PHO(200), TO(200), U0(402), V0(200), PHRA(200),
     + PHRB(200), PHIA(200), PHIB(200), TRA(200), TRB(200), TIA(200),
     + TIB(200), URA(200), URB(200), UIA(200), UIB(200), VRA(200),
     + VRB(200), VIA(200), VIB(200),PHR(200), PHI(200), TR(200),
     + TI(200), UR(200), UI(200), VR(200), VI(200), FL(200), FG(200),
     + DFL(200), DFG(200), T(8), QPHO(200), QTO(200),  QPHRA(200),
     + QPHRB(200), QPHIA(200), QPHIB(200),  QTRA(200), QTRB(200),
     + QTIA(200), QTIB(200), QURA(200), QURB(200), QUIA(200),
     + QUIB(200), QVRA(200), QVRB(200), QVIA(200), QVIB(200)
      COMMON DS,WUL,D,ALAM0,ALAM1,ALAM2,BLAM1,
     1BLAM2,AGAM0,AGAM1,AGAM2,BGAM1,BGAM2,
     2OM,ULM,APP,C,PHO,DFL,DFG,K,I,TO,PHR,PHI,FL,FG
      EXTERNAL DAUX,DARN
      READ (5,11) TITLE,
     + BLO, BDO, BW,
     + BLM, DBL, BDM, DBD, OM,
     + BM, BMS, BMH,
     + WUL, ULM, DS, N,
     + C, DN, CH, CD, AMP,
     + ALAM0, ALAM1, ALAM2, BLAM1, BLAM2,
     + AGAM0, AGAM1, AGAM2, BGAM1, BGAM2
C    WRITE TITLE AND INPUT DATA
      WRITE (6,21) TITLE
      WRITE (6,22) BLO, BDO, BW
      WRITE (6,23) BLM, DBL, BDM, DBD, OM
      WRITE (6,24) BM, BMS, BMH
      WRITE (6,25) WUL, ULM, DS, N
      WRITE (6,26) C, DN, CH, CD, AMP
      WRITE (6,27) ALAM0, ALAM1, ALAM2, BLAM1, BLAM2
      WRITE (6,28) AGAM0, AGAM1, AGAM2, BGAM1, BGAM2
C    INITIALIZE
      D = 0.5*DN*C*C*CH*CD
      AMC = 3.14159*CH*CH*DN*AMP/4.0
      APP  =   1.0 + AMC/ULM
      CX0 = 0.0
      CY0 = 0.0
      CXR = 0.0
      CXI = 0.0
      CYR = 0.0
      CYI = 0.0
      KA  =  N-1
      IA  =  N-1
C    SOLVE STEADY-STATE PROBLEM
      ERR=.00001
      ERA=.001
      CALL ATA(BDO,YK)
      PHOB  =  ATAN2(BLO-BW, -YK)
      TOB = SQRT(BDO*BDO +(BLO-BW)*(BLO-BW))
      WRITE (6,31)
      WRITE (6,32)
      PHO(1) = PHOB
      TO(1) = TOB
C    MULTIPLY TO FIND VELOCITY COMPONENTS
      DO 101  K = 1,N
```

```
      UO(K) = C*SIN(PHO(K))
      VO(K) = C*COS(PHO(K))
      J  =  - K
      WRITE (6,33) PHO(K), TO(K), UO(K), VO(K), CXO, CYO, J
      IF (K.EQ.N)  GO TO 101
C   INTEGRATE TO FIND TENSION, ANGLE, AND CABLE CONFIGURATION
      T(1)=TO(K)
      T(2)=PHO(K)
      T(3)  =  CXO
      T(4)  =  CYO
      FIRST=.0
      ALS=FLOAT(K)*DS
      CALL KUTMER(4,ALS,T,ERR,DS,FIRST,HCX,ERA,DAUX)
      TO(K+1)=T(1)
      PHO(K+1)=T(2)
      CXO  =  T(3)
      CYO  =  T(4)
101   CONTINUE
C   SOLVE DYNAMIC PROBLEM
      WRITE (6,34)
      WRITE (6,35)
C   REVERSE LISTS OF  PHO  AND  TO
      DO  301  I  =  1, N
      QPHO(I)  =  PHO(I)
      QTO(I)  =  TO(I)
301   CONTINUE
      DO  302  I  =  1, N
      L  =  N+1-I
      PHO(I)  =  QPHO(L)
      TO(I)  =  QTO(L)
302   CONTINUE
C   COMPUTE VISCOUS-FORCE TERMS ALONG CABLE
      DO 201  I = 1, IA
      FL(I) = D*(+ALAMO-ALAM1*COS(PHO(I))+ALAM2*COS(2.0*PHO(I))+
     + BLAM1*SIN(PHO(I))-BLAM2*SIN(2.0*PHO(I)))
      FG(I) = D*(-AGAMO+AGAM1*COS(PHO(I))-AGAM2*COS(2.0*PHO(I))
     + -BGAM1*SIN(PHO(I))+BGAM2*SIN(2.0*PHO(I)))
      DFL(I) = D*(+ALAM1*SIN(PHO(I))-2.0*ALAM2*SIN(2.0*PHO(I))
     + +BLAM1*COS(PHO(I))-2.0*BLAM2*COS(2.0*PHO(I)))
      DFG(I) = D*(-AGAM1*SIN(PHO(I))+2.0*AGAM2*SIN(2.0*PHO(I))
     + -BGAM1*COS(PHO(I))+2.0*BGAM2*COS(2.0*PHO(I)))
201   CONTINUE
C   COMPUTE MODAL SOLUTIONS
C   COMPUTE  A -MODE SOLUTION
C   SET VELOCITY COMPONENTS EQUAL TO ZERO AT BOTTOM
      URA(1)  =  0.0
      UIA(1) = 0.0
      VRA(1) = 0.0
      VIA(1) = 0.0
C   ASSIGN VALUES TO ANGLE AND TENSION AT BOTTOM
      PHRA(1)  =  0.001
      PHIA(1)  =  0.0
      TRA(1)  =  0.0
      TIA(1)  =  0.0
C   FIND VELOCITY COMPONENTS, ANGLE, AND TENSION BY INTEGRATING UP
```

```
C     THE CABLE
      DO 202  I = 1, IA
      T(1)=PHRA(I)
      T(2)=PHIA(I)
      T(3)=TRA(I)
      T(4)=TIA(I)
      T(5)=URA(I)
      T(6)=UIA(I)
      T(7)=VRA(I)
      T(8)=VIA(I)
      FIRST=.0
      ALS=FLOAT(I)*DS
      CALL KUTMER(8,ALS,T,ERR,DS,FIRST,HCX,ERA,DARN)
      PHRA(I+1)=T(1)
      PHIA(I+1)=T(2)
      TRA(I+1)=T(3)
      TIA(I+1)=T(4)
      URA(I+1)=T(5)
      UIA(I+1)=T(6)
      VRA(I+1)=T(7)
      VIA(I+1)=T(8)
202   CONTINUE
C     COMPUTE  B -MODE SOLUTION
C     SET VELOCITY COMPONENTS EQUAL TO ZERO AT BOTTOM
      URB(1) = 0.0
      UIB(1) = 0.0
      VRB(1)   =  0.0
      VIB(1) = 0.0
C     ASSIGN VALUES TO ANGLE AND TENSION AT BOTTOM
      PHRB(1)    =  0.0
      PHIB(1)    =  0.0
      TRB(1)   =  0.1
      TIB(1)   =  0.0
C     FIND VELOCITY COMPONENTS, ANGLE, AND TENSION BY INTEGRATING UP
C     THE CABLE
      DO 203  I = 1, IA
      T(1)=PHRB(I)
      T(2)=PHIB(I)
      T(3)=TRB(I)
      T(4)=TIB(I)
      T(5)=URB(I)
      T(6)=UIB(I)
      T(7)=VRB(I)
      T(8)=VIB(I)
      FIRST=.0
      ALS=FLOAT(I)*DS
      CALL KUTMER(8,ALS,T,ERR,DS,FIRST,HCX,ERA,DARN)
      PHRB(I+1)=T(1)
      PHIB(I+1)=T(2)
      TRB(I+1)=T(3)
      TIB(I+1)=T(4)
      URB(I+1)=T(5)
      UIB(I+1)=T(6)
      VRB(I+1)=T(7)
      VIB(I+1)=T(8)
```

```
203   CONTINUE
C   REVERSE LISTS OF  PHO ,  TO , AND MODAL SOLUTIONS
      DO  303  K  =  1, N
      QPHO(K)  =  PHO(K)
      QTO(K)  =  TO(K)
      QURA(K)  =  URA(K)
      QUIA(K)  =  UIA(K)
      QVRA(K)  =  VRA(K)
      QVIA(K)  =  VIA(K)
      QPHRA(K)  =  PHRA(K)
      QPHIA(K)  =  PHIA(K)
      QTRA(K)  =  TRA(K)
      QTIA(K)  =  TIA(K)
      QURB(K)  =  URB(K)
      QUIB(K)  =  UIB(K)
      QVRB(K)  =  VRB(K)
      QVIB(K)  =  VIB(K)
      QPHRB(K)  =  PHRB(K)
      QPHIB(K)  =  PHIB(K)
      QTRB(K)  =  TRB(K)
      QTIB(K)  =  TIB(K)
303   CONTINUE
      DO  304  K  =  1, N
      M  =  N+1-K
      PHO(K)  =  QPHO(M)
      TO(K)  =  QTO(M)
      URA(K)  =  QURA(M)
      UIA(K)  =  QUIA(M)
      VRA(K)  =  QVRA(M)
      VIA(K)  =  QVIA(M)
      PHRA(K)  =  QPHRA(M)
      PHIA(K)  =  QPHIA(M)
      TRA(K)  =  QTRA(M)
      TIA(K)  =  QTIA(M)
      URB(K)  =  QURB(M)
      UIB(K)  =  QUIB(M)
      VRB(K)  =  QVRB(M)
      VIB(K)  =  QVIB(M)
      PHRB(K)  =  QPHRB(M)
      PHIB(K)  =  QPHIB(M)
      TRB(K)  =  QTRB(M)
      TIB(K)  =  QTIB(M)
304   CONTINUE
C   SUPERIMPOSE MODAL SOLUTIONS
C   COMPUTE FORCE COMPONENTS AT TOP
      BDR  =  BDM*COS(BDB)
      BDI  =  BDM*SIN(BDB)
      BLR  =  BLM*COS(DBL)
      BLI  =  BLM*SIN(DBL)
      PR  =  - BDR*COS(PHO(1)) + BLR*SIN(PHO(1))
      PI  =  - BDI*COS(PHO(1)) + BLI*SIN(PHO(1))
      QR  =  BDR*SIN(PHO(1)) + BLR*COS(PHO(1))
      QI  =  BDI*SIN(PHO(1)) + BLI*COS(PHO(1))
      PAR  =  TRA(1) - OM*(+(BMH-BMS)*UIA(1)*COS(PHO(1))*SIN(PHO(1))
     + -(BM+BM [illegible] OS(PHO(1))*COS(PHO(1))+BMH*SIN(PHO(1))*SIN(PHO(1)))*
```

```
     + VIA(1))
       PAI  =  TIA(1) - OM*(-(BMH-BMS)*URA(1)*COS(PHO(1))*SIN(PHO(1))
     + +(BM+BMS*COS(PHO(1))*COS(PHO(1))+BMH*SIN(PHO(1))*SIN(PHO(1)))*
     + VRA(1))
       PBR  =  TRB(1) - OM*(+(BMH-BMS)*UIB(1)*COS(PHO(1))*SIN(PHO(1))
     + -(BM+BMS*COS(PHO(1))*COS(PHO(1))+BMH*SIN(PHO(1))*SIN(PHO(1)))*
     + VIB(1))
       PBI  =  TIB(1) - OM*(-(BMH-BMS)*URB(1)*COS(PHO(1))*SIN(PHO(1))
     + +(BM+BMS*COS(PHO(1))*COS(PHO(1))+BMH*SIN(PHO(1))*SIN(PHO(1)))*
     + VRB(1))
       QAR  =  TO(1)*PHRA(1)-OM*(+(BM+BMS*SIN(PHO(1))*SIN(PHO(1))+
     + BMH*COS(PHO(1))*COS(PHO(1)))*UIA(1)-(BMH-BMS)*VIA(1)*COS(PHO(1))
     + *SIN(PHO(1)))
       QAI  =  TO(1)*PHIA(1)-OM*(-(BM+BMS*SIN(PHO(1))*SIN(PHO(1))+
     + BMH*COS(PHO(1))*COS(PHO(1)))*URA(1)+(BMH-BMS)*VRA(1)*COS(PHO(1))
     + *SIN(PHO(1)))
       QBR  =  TO(1)*PHRB(1)-OM*(+(BM+BMS*SIN(PHO(1))*SIN(PHO(1))+
     + BMH*COS(PHO(1))*COS(PHO(1)))*UIB(1)-(BMH-BMS)*VIB(1)*COS(PHO(1))
     + *SIN(PHO(1)))
       QBI  =  TO(1)*PHIB(1)-OM*(-(BM+BMS*SIN(PHO(1))*SIN(PHO(1))+
     + BMH*COS(PHO(1))*COS(PHO(1)))*URB(1)+(BMH-BMS)*VRB(1)*COS(PHO(1))
     + *SIN(PHO(1)))
       DENOMR  =  PAR*QBR-PAI*QBI-PBR*QAR+PBI*QAI
       DENOMI  =  PAI*QBR+PAR*QBI-PBI*QAR-PBR*QAI
       ANUMR   =  P R*QBR-P I*QBI-PBR*Q R+PBI*Q I
       ANUMI   =  P I*QBR+P R*QBI-PBI*Q R-PBR*Q I
       BNUMR   =  PAR*Q R-PAI*Q I-P R*QAR+P I*QAI
       BNUMI   =  PAI*Q R+PAR*Q I-P I*QAR-P R*QAI
       DENOM2  =  DENOMR*DENOMR + DENOMI*DENOMI
       CAR   =   (ANUMR*DENOMR + ANUMI*DENOMI)/DENOM2
       CAI   =  (-ANUMR*DENOMI + ANUMI*DENOMR)/DENOM2
       CBR   =   (BNUMR*DENOMR + BNUMI*DENOMI)/DENOM2
       CBI   =  (-BNUMR*DENOMI + BNUMI*DENOMR)/DENOM2
C     COMPUTE VELOCITY COMPONENTS, ANGLE, AND TENSION ON CABLE
       DO 204  K = 1, N
       UR(K)   =  CAR*URA(K) - CAI*UIA(K) + CBR*URB(K) - CBI*UIB(K)
       UI(K)   =  CAI*URA(K) + CAR*UIA(K) + CBI*URB(K) + CBR*UIB(K)
       VR(K)   =  CAR*VRA(K) - CAI*VIA(K) + CBR*VRB(K) - CBI*VIB(K)
       VI(K)   =  CAI*VRA(K) + CAR*VIA(K) + CBI*VRB(K) + CBI*VIB(K)
       PHR(K)   =  CAR*PHRA(K) - CAI*PHIA(K) + CBR*PHRB(K) - CBI*PHIB(K)
       PHI(K)   =  CAI*PHRA(K) + CAR*PHIA(K) + CBI*PHRB(K) + CBR*PHIB(K)
       TR(K)   =  CAR*TRA(K) - CAI*TIA(K) + CBR*TRB(K) - CBI*TIB(K)
       TI(K)   =  CAI*TRA(K) + CAR*TIA(K) + CBI*TRB(K) + CBR*TIB(K)
C     COMPUTE PHYSICAL VELOCITY COMPONENTS ON CABLE
       URP   =  C*PHR(K)*COS(PHO(K)) + UR(K)
       UIP   =  C*PHI(K)*COS(PHO(K)) + UI(K)
       VRP   =  - C*PHR(K)*SIN(PHO(K)) + VR(K)
       VIP   =  - C*PHI(K)*SIN(PHO(K)) + VI(K)
C     COMPUTE MAGNITUDE AND PHASE OF VELOCITY COMPONENTS, ANGLE, AND
C     TENSION ON CABLE
       UM   =  SQRT(URP*URP + UIP*UIP)
       CALL ATA(URP, YK)
       DU   =  ATAN2(UIP, YK)
       VM   =  SQRT(VRP*VRP + VIP*VIP)
       CALL ATA(VRP, YK)
```

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      DV   =  ATAN2(VIP, YK)
      PHM  =  SQRT(PHR(K)*PHR(K) + PHI(K)*PHI(K))
      CALL ATA(PHR(K), YK)
      DPH  =  ATAN2(PHI(K), YK)
      TM   =  SQRT(TR(K)*TR(K) + TI(K)*TI(K))
      CALL ATA(TR(K), YK)
      DT   =  ATAN2(TI(K), YK)
      J  =  - K
      WRITE (6,36) PHM, DPH, TM, DT, UM, DU, VM, DV, J
204   CONTINUE
      WRITE (6,37)
C   COMPUTE MAGNITUDE AND PHASE OF DYNAMIC CABLE CONFIGURATION
      DO 205  K = 1,N
      CXM = SQRT(CXR*CXR+CXI*CXI)
      CALL ATA(CXR, YK)
      DCX  =  ATAN2(CXI, YK)
      CYM = SQRT(CYR*CYR+CYI*CYI)
      CALL ATA(CYR, YK)
      DCY  =  ATAN2(CYI, YK)
      J  =  - K
      WRITE (6,38) CXM, DCX, CYM, DCY, J
      IF (K.EQ. N) GO TO 205
      CXR  =  PHR(K)*SIN(PHO(K))*DS
      CXI  =  PHI(K)*SIN(PHO(K))*DS
      CYR  = -PHR(K)*COS(PHO(K))*DS
      CYI  = -PHI(K)*COS(PHO(K))*DS
205   CONTINUE
11    FORMAT ( 10X,  A6    / 10X, 3F10.5/ 10X, 5F10.5/ 10X, 3F10.5/
     +  10X, 3F10.5, I10/ 10X, 5F10.5 / 10X, 5F10.5/ 10X, 5F10.5)
21    FORMAT (1H1, 10X, 53HDEEPLY SUBMERGED SUBMARINE TOWING A FLOAT IN
     +A SEAWAY / 1H-, 13X,  A6///)
22    FORMAT ( 1X, 8HBLO     = , F10.5, 4X, 8HBDO     = , F10.5, 4X,
     + 8HBW      = , F10.5 //)
23    FORMAT ( 1X, 8HBLM     = , F10.5, 4X, 8HDBL     = , F10.5, 4X,
     + 8HBDM     = , F10.5, 4X, 8HDBD     = , F10.5, 4X, 8HOM      = ,
     + F10.5 //)
24    FORMAT ( 1X, 8HBM      = , F10.5, 4X, 8HBMS     = , F10.5, 4X,
     + 8HBMH     = , F10.5 //)
25    FORMAT ( 1X, 8HWUL     = , F10.5, 4X, 8HULM     = , F10.5, 4X,
     + 8HDS      = , F10.5, 4X, 8HN       = , I10 //)
26    FORMAT ( 1X, 8HC       = , F10.5, 4X, 8HDN      = , F10.5, 4X,
     + 8HCH      = , F10.5, 4X, 8HCD      = , F10.5, 4X, 8HAMP     = ,F10.5//)
27    FORMAT ( 1X, 8HALAM0 = ,F10.5, 4X, 8HALAM1 = , F10.5,4X,
     + 8HALAM2 = , F10.5, 4X, 8HBLAM1 = , F10.5, 4X, 8HBLAM2 = ,
     + F10.5 //)
28    FORMAT ( 1X, 8HAGAM0 = , F10.5, 4X, 8HAGAM1 = , F10.5, 4X,
     + 8HAGAM2 = , F10.5, 4X, 8HBGAM1 = , F10.5, 4X, 8HBGAM2 = ,
     + F10.5 //)
31    FORMAT ( 22H1STEADY-STATE SOLUTION)
32    FORMAT ( 1H0, 1X, 6HPHO(J), 10X, 6HTO(J) , 10X, 6HUO(J) , 10X,
     + 6HVO(J) , 10X, 6HCXO(J), 10X, 6HCYO(J), 10X, 1HJ  //)
33    FORMAT ( 2X, F9.6, 7X, F13.6, 3X, 2( F11.6, 5X), 2(F13.6, 3X), I4)
34    FORMAT ( 17H1DYNAMIC SOLUTION)
35    FORMAT ( 1H0, 1X, 6HPHM(J), 9X, 6HDPH(J), 9X, 6HTM(J) , 9X,
     + 6HDT(J) , 9X, 6HUM(J) , 9X, 6HDU(J) , 9X, 6HVM(J) , 9X, 6HDV(J) ,
```

```
     + 9X, 1HJ //)
36    FORMAT (      2X, 2( F9.6, 6X), F12.6, 3X, F9.6, 6X, 2(F10.6, 5X,
     + F9.6, 6X), I4)
37    FORMAT ( 1H1, 1X, 6HCXM(J), 10X, 6HDCX(J), 10X, 6HCYM(J), 10X,
     + 6HDCY(J), 10X, 1HJ //)
38    FORMAT ( 2X, 2( F12.6, 4X, F9.6, 7X), I4)
      STOP
      END


      SUBROUTINE  ATA(X,YK)
C     THIS SUBROUTINE PREVENTS  ATAN2  FROM DIVIDING BY ZERO WHEN
C     COMPUTING THE ARCTANGENTS OF  PI/2  AND  3*PI/2
      ER = 0.00001
      IF (ABS(X).GT.ER) GO TO 10
      YK = ER
      RETURN
10    YK = X
      RETURN
      END


      SUBROUTINE KUTMER(N,T,Y0,EPS,H,FIRST,HCX,A,DAUX)
C     THIS SUBROUTINE PERFORMS FOURTH-ORDER KUTTA-MERSON INTEGRATIONS
      DIMENSION Y0(10),Y1(10),Y2(10),F0(10),F1(10),F2(10)
      IF(FIRST)20,10,20
   10 HC=H                                                              KUTM0050
      IPLOC=1                                                           KUTM0060
      FIRST=1.                                                          KUTM0070
   20 LOC=0                                                             KUTM0080
       HCX=HC                                                           KUTM0090
   30 CALL DAUX(T,Y0,F0)                                                KUTM0100
   39 DO 40 I=1,N                                                       KUTM0110
   40 Y1(I)=Y0(I)+(HC/3.)*F0(I)                                         KUTM0120
      CALL DAUX(T+HC/3.,Y1,F1)                                          KUTM0130
      DO 50 I=1,N                                                       KUTM0140
   50 Y1(I)=Y0(I)+(HC/6.)*F0(I)+(HC/6.)*F1(I)                           KUTM0150
      CALL DAUX(T+HC/3.,Y1,F1)                                          KUTM0160
      DO 60 I=1,N                                                       KUTM0170
   60 Y1(I)=Y0(I)+HC/8.*F0(I)+.375*HC*F1(I)                             KUTM0180
      CALL DAUX(T+HC/2.,Y1,F2)                                          KUTM0190
      DO 70 I=1,N                                                       KUTM0200
   70 Y1(I)=Y0(I)+HC/2.*F0(I)-1.5*HC*F1(I)+2.*HC*F2(I)                  KUTM0210
      CALL DAUX(T+HC,Y1,F1)                                             KUTM0220
      DO 80 I=1,N                                                       KUTM0230
   80 Y2(I)=Y0(I)+HC/6.*F0(I)+.66666667*HC*F2(I)+(HC/6.)*F1(I)          KUTM0240
      INC=0                                                             KUTM0250
      DO 110 I=1,N                                                      KUTM0260
      ZZZ=ABS(Y1(I))-A                                                  KUTM0270
       IF(ZZZ) 85,87,87                                                 KUTM0280
   85 ERROR=ABS(.2*(Y1(I)-Y2(I)))                                       KUTM0290
       IF(ERROR-A)100,100,90                                            KUTM0300
   87 ERROR=ABS(.2-.2*Y2(I)/Y1(I))                                      KUTM0310
      IF(ERROR-EPS)100,100,90                                           KUTM0320
   90 X=128.*ABS(HC)-ABS(H)                                             KUTM0330
       IF(X) 91,95,95                                                   KUTM0340
   91 WRITE(6,92) T,ERROR                                               KUTM0350
   92 FORMAT(21H RELATIVE ERROR AT T= 1P1E12.3,3HIS F10.6    )          KUTM0360
       FIRST = 2.                                                       KUTM0370
       RETURN                                                           KUTM0380
   95 HC=HC/2.                                                          KUTM0390
      IPLOC=2 *IPLOC                                                    KUTM0400
      LOC=2 *LOC                                                        KUTM0410
       HCX=HC                                                           KUTM0420
      GO TO 30                                                          KUTM0430
  100 IF(ERROR*64.-EPS)110,110,101                                      KUTM0440
  101 INC=1                                                             KUTM0450
  110 CONTINUE                                                          KUTM0460
  111 T=T+HC                                                            KUTM0470
      DO 112 I=1,N                                                      KUTM0480
  112 Y0(I)=Y2(I)                                                       KUTM0490
      LOC=LOC+1                                                         KUTM0500
       IF(LOC-IPLOC)120,210,210                                         KUTM0510
  120 IF(INC)210,130,210                                                KUTM0520
  130 IF(LOC-(LOC/2)*2)210,140,210                                      KUTM0530
  140 IF(IPLOC-1)210,210,200                                            KUTM0540
  200 HC=2.*HC                                                          KUTM0550
```

```
      LOC=LOC/2                                                         KUTM0560
      IPLOC=IPLOC/2                                                     KUTM0570
  210 IF(IPLOC-LOC)30,220,30                                            KUTM0580
  220 RETURN                                                            KUTM0590
      END                                                               KUTM0600
```

```
      SUBROUTINE DAUX(Z,TX,F)
C     THIS SUBROUTINE PROVIDES THE INTEGRANDS FOR  KUTMER  WHEN IT
C     COMPUTES THE STEADY-STATE TENSION, ANGLE, AND CABLE CONFIGURATION
      DIMENSION  TX(8), F(8),PHO(200),DFL(200),DFG(200),TO(200),
     + PHR(200), PHI(200)
      COMMON DS,WUL,D,ALAMO,ALAM1,ALAM2,BLAM1,
     1BLAM2,AGAMO,AGAM1,AGAM2,BGAM1,BGAM2,
     2OM,ULM,APP,C,PHO,DFL,DFG,K,I,TO,PHR,PHI
      TX1=TX(1)
      TX2=TX(2)
      C1=COS(TX2)
      S1=SIN(TX2)
      C2=COS(2.*TX2)
      S2=SIN(2.*TX2)
      FL=D*(ALAMO-ALAM1*C1+ALAM2*C2+BLAM1*S1-BLAM2*S2)
      FG=D*(-AGAMO+AGAM1*C1-AGAM2*C2-BGAM1*S1+BGAM2*S2)
      IF(TX1.EQ.0.) GO TO 50
      F(2)=-(WUL*C1-FL)/TX1
      F(3)  =  - C1
      F(4)  =  - S1
    7 F(1)=-(WUL*S1+FG)
      RETURN
   50 F(2)=0.0
      GO TO 7
      END
```

```
      SUBROUTINE DARN(Z,T,F)
C     THIS SUBROUTINE PROVIDES THE INTEGRANDS FOR  KUTMER  WHEN IT
C     COMPUTES THE DYNAMIC ANGLE, TENSION, AND VELOCITY COMPONENTS
      DIMENSION  T(8), F(8),PHO(200),DFL(200),DFG(200),TO(200),
     + FL(200),FG(200),PHR(200),PHI(200)
      COMMON DS,WUL,D,ALAMO,ALAM1,ALAM2,BLAM1,
     1BLAM2,AGAMO,AGAM1,AGAM2,BGAM1,BGAM2,
     2OM,ULM,APP,C,PHO,DFL,DFG,K,I,TO,PHR,PHI,FL,FG
      C1=COS(PHO(I))
      S1=SIN(PHO(I))
      IF(TO(I).EQ.0.0) GO TO 50
      F(1)=(-OM*ULM*(APP*T(6)+(APP-1.0)*C*T(2)*C1)-T(3)*(PHO(I+1)-PHO(I)
     +)/DS-(T(1)+(T(5)*C1-T(7)*S1)/C)*DFL(I)
     +                  -2.0*FL(I)*(T(5)*S1+T(7)*C1)/C-WUL*T(1)*S1)/TO(I)
      F(2)=( OM*ULM*(APP*T(5)+(APP-1.0)*C*T(1)*C1)-T(4)*(PHO(I+1)-PHO(I)
     +)/DS-(T(2)+(T(6)*C1-T(8)*S1)/C)*DFL(I)
     +                  -2.0*FL(I)*(T(6)*S1+T(8)*C1)/C-WUL*T(2)*S1)/TO(I)
    7 F(3)=( OM*ULM*(T(8)+(APP-1.0)*C*T(2)*S1)+(T(1)+(T(5)*C1-
     + T(7)*S1)/C)*DFG(I)+2.0*FG(I)*
     + (T(5)*S1+T(7)*C1)/C+WUL*T(1)*C1)
      F(4)=(-OM*ULM*(T(7)+(APP-1.0)*C*T(1)*S1)+(T(2)+(T(6)*C1-
     + T(8)*S1)/C)*DFG(I)+2.0*FG(I)*
     + (T(6)*S1+T(8)*C1)/C+WUL*T(2)*C1)
      F(5)=-OM*T(2)+T(7)*(PHO(I+1)-PHO(I))/DS
      F(6)= OM*T(1)+T(8)*(PHO(I+1)-PHO(I))/DS
      F(7)=-T(5)*(PHO(I+1)-PHO(I))/DS
      F(8)=-T(6)*(PHO(I+1)-PHO(I))/DS
      RETURN
   50 F(1)=0.0
      F(2)=0.0
      GO TO 7
      END
```

DEEPLY SUBMERGED SUBMARINE TOWING A FLOAT IN A SEAWAY
OMFLO

BLO	=	2122.00000	BDO	=	850.00000	BW	=	0.00000						
BLM	=	0.00000	DBL	=	0.00000	BDM	=	1.00000	DBD	=	0.00000	OM	=	3.04000
BM	=	59.00000	BMS	=	1.55200	BMH	=	14.58000						
WUL	=	.68000	ULM	=	.02730	DS	=	10.00000	N	=	20			
C	=	23.60000	DN	=	1.99040	CH	=	.06250	CD	=	1.90000	AMP	=	1.00000
ALAMO	=	.47500	ALAM1	=	0.00000	ALAM2	=	-.47500	BLAM1	=	.05000	BLAM2	=	0.00000
AGAMO	=	0.00000	AGAM1	=	.05000	AGAM2	=	0.00000	BGAM1	=	0.00000	BGAM2	=	0.00000

STEADY-STATE SOLUTION

PHO(J)	TO(J)	UO(J)	VO(J)	CXO(J)	CYO(J)	J
1.951790	2285.909884	21.907775	-8.775499	0.000000	0.000000	-1
2.176047	2295.620872	19.407684	-13.427651	4.762587	-8.769194	-2
2.348787	2311.473163	16.810882	-16.563643	11.163456	-16.436043	-3
2.478824	2331.591216	14.521119	-18.603685	18.645109	-23.060514	-4
2.577273	2354.629288	12.622241	-19.940888	26.830685	-28.797703	-5
2.653124	2379.697744	11.074867	-20.840041	35.482798	-33.806982	-6
2.712791	2406.216544	9.812436	-21.463367	44.453089	-38.223331	-7
2.760696	2433.801924	8.773380	-21.908624	53.647124	-42.154053	-8
2.799884	2462.193588	7.908299	-22.235530	63.003111	-45.682888	-9
2.832483	2491.210101	7.179372	-22.481473	72.479422	-48.875170	-10
2.860004	2520.721610	6.558017	-22.670519	82.047210	-51.782250	-11
2.883543	2550.632873	6.022614	-22.818592	91.685964	-54.444919	-12
2.903908	2580.872475	5.556688	-22.936504	101.380755	-56.895950	-13
2.921708	2611.385792	5.147557	-23.031775	111.120492	-59.161981	-14
2.937408	2642.130296	4.785350	-23.109747	120.896773	-61.264901	-15
2.951367	2673.072349	4.462296	-23.174294	130.703132	-63.222883	-16
2.963870	2704.184960	4.172213	-23.228272	140.534516	-65.051157	-17
2.975142	2735.446196	3.910129	-23.273824	150.386921	-66.762605	-18
2.985364	2766.838026	3.672009	-23.312579	160.257138	-68.368206	-19
2.994686	2798.345473	3.454545	-23.345794	170.142563	-69.877391	-20

DYNAMIC SOLUTION

PHM(J)	DPH(J)	TM(J)	DT(J)	UM(J)	DU(J)	VM(J)	DV(J)	J
.000248	1.698381	.750140	.518199	.007897	-1.937456	.009741	1.301676	-1
.000115	2.359663	.747028	.527085	.005429	-.936508	.014934	1.334075	-2
.000059	3.063075	.743954	.529894	.004678	-.243825	.009626	1.321030	-3
.000036	-2.570909	.742234	.530370	.004048	.175187	.004912	1.287957	-4
.000025	-2.065947	.741489	.530299	.003393	.438684	.002237	1.219584	-5
.000018	-1.065041	.741257	.530246	.002799	.605454	.000953	1.078221	-6
.000013	-.913533	.741259	.530309	.002312	.704423	.000406	.795728	-7
.000009	-.717213	.741366	.530456	.001933	.751930	.000207	.361589	-8
.000007	-.515460	.741523	.530642	.001648	.759950	.000140	-.002347	-9
.000005	-.350383	.741707	.530827	.001436	.738878	.000105	-.165939	-10
.000005	-.242116	.741910	.530985	.001273	.697815	.000078	-.192575	-11
.000004	-.190552	.742124	.531101	.001144	.644405	.000055	-.137305	-12
.000004	-.186901	.742342	.531167	.001030	.584429	.000037	-.006801	-13
.000004	-.221316	.742554	.531178	.000922	.522310	.000023	.257606	-14
.000005	-.283900	.742750	.531137	.000811	.462343	.000016	.779362	-15
.000005	-.364042	.742920	.531050	.000689	.410810	.000016	.995966	-16
.000006	-.450521	.743052	.530930	.000555	.380710	.000019	1.092946	-17
.000006	-.532779	.743138	.530792	.000407	.407028	.000023	1.154436	-18
.000007	-.602308	.743170	.530654	.000258	.615428	.000025	1.191808	-19
.000008	-.652989	.743146	.530539	.000181	1.404510	.000027	1.209021	-20

CXM(J)	DCX(J)	CYM(J)	DCY(J)	J
0.000000	0.000000	0.000000	0.000000	-1
.002306	1.698381	.000924	1.698381	-2
.000948	2.359663	.000656	2.359663	-3
.000421	3.063075	.000415	3.063075	-4
.000224	-2.570909	.000287	-2.570909	-5
.000135	-2.065947	.000213	-2.065947	-6
.000085	-1.693879	.000161	-1.6938[illegible]	-7
.000055	-1.387275	.000119	-1.406288	-8
.000035	-1.163365	.000088	-1.163365	-9
.000024	-.939187	.000066	-.939187	-10
.000017	-.729205	.000052	-.729205	-11
.000013	-.557606	.000045	-.557606	-12
.000011	-.457430	.000042	-.461557	-13
.000010	-.418907	.000042	-.457120	-14
.000010	-.456233	.000043	-.534646	-15
.000009	-.534252	.000046	-.673476	-16
.000010	-.624314	.000050	-.853303	-17
.000010	-.707460	.000055	-1.058458	-18
.000010	-.773746	.000061	-1.278290	-19
.000011	-.819087	.000068	-1.424606	-20

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